Test of the Einstein relation in relativistic collisions

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Abstract

As an important ingredient of a nonequilibrium-statistical description of relativistic collisions, the validity of a weak-coupling dissipation-fluctuation theorem (Einstein relation) in rapidity space is tested from SIS via AGS to SPS energies. The theorem relates the rapidity relaxation time to the diffusion coefficient via the temperature of the equilibrium distribution. Whereas it is fulfilled at low SIS-energies, progressively larger deviations rising to an order of magnitude are found at AGS- and SPS-energies. The deviations are related to the strong-coupling character of the system which may also induce non-equilibrium quark-gluon plasma formation.

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1 Introduction

Relativistic heavy-ion collisions [1] offer a unique possibility to study equilibration processes in microscopic many-body systems. In particular, rapidity distributions for participant protons are valuable indicators for the time-evolution of the reactions. They can be used to investigate the approach to thermal equilibrium in the interaction region.

Isotropic thermal models fail to reproduce the broad, often flat or even dipped rapidity spectra that have been observed at AGS and SPS energies. Hence, these models have been supplemented [2] by collective expansion and anisotropic flow schemes, or by a superposition of isotropically decaying thermal sources (fireballs) [3] to fit the data [4]. The broad rapidity distributions for participant protons that have been observed may, however, also indicate nonequilibrium properties of the system: Due to the short interaction time, thermal equilibrium in the interaction region is not completely reached even in case of central collisions of heavy systems such as Pb+Pb. A nonequilibrium-statistical model based on a linear Fokker-Planck equation (FPE) [5] has therefore been introduced [6] to describe the gradual approach to local thermal equilibrium in the interaction zone.

This relaxation process occurs as a consequence of random collisions and creation of secondaries. The nonequilibrium rapidity spectrum for protons emerges analytically from the model. A similar relativistic diffusion model (RDM) has previously been used to evaluate and predict transverse energy distributions (integrated over all particle species) analytically [7]. The model allows, in particular, to interpret flat or even double-peaked proton rapidity spectra that have been observed in heavy relativistic systems [8 - 11] as being due to incomplete thermalization. As an important ingredient of such a nonequilibrium-statistical description, the validity of a weak-coupling dissipation-fluctuation theorem is tested here from SIS- via AGS- to SPS-energies.

2 Relativistic diffusion model

In the relativistic diffusion model for rapidity spectra of participants, the initial projectile rapidity

$$y_1 = \frac{1}{2} \ln \left(\frac{E_L + P_L}{E_L - P_L} \right) \tag{1}$$

with laboratory energy E_L and momentum P_L, as well as the initial target rapidity y_2 (= 0 for fixed-target experiments) relaxes towards the equilibrium value y_{eq} given by the center-of-mass rapidity due to collisions, particle creations and decays. The discussion is restricted to participant protons which (as compared to produced particles) exhibit the nonequilibrium properties most clearly. Transforming to

$$Y = \frac{2y}{y_1} - 1 \tag{2}$$

such that Y = 1 for y = y_1, and Y = -1 for y = 0, the initial conditions of the time-dependent rapidity distribution functions for projectile (target) participants are $R_{1,2}(Y,0) = \delta(Y \mp 1)$. Here I neglect a small initial broadening $Y \approx 0.07$ which is due to the Fermi momentum.

A partial differential equation for the distribution function R (Y,t) of the macroscopic variable Y should account for both the shift of the mean value $\langle Y \rangle$ towards equilibrium Y_{eq} and the corresponding broadening in time. The Fokker-Planck equation successfully describes similar nonequilibrium processes in many areas of physics [5] - starting with the theory of Brownian motion, where it has been used to model both the velocity- and the coordinate distribution. In relativistic systems, however, Lorentz-invariant kinematical variables have to be introduced. In particular, the rapidity replaces the velocity to describe the motion parallel to the beam direction, and the FPE for the probability distribution of the rapidity becomes

$$\frac{\partial R(Y,t)}{\partial t} = \frac{1}{\tau_Y} \frac{\partial}{\partial Y}[(Y - Y_{eq})R(Y,t)] + D_Y \frac{\partial^2}{\partial Y^2} R(Y,t). \tag{3}$$

Two transport coefficients determine the time evolution of the rapidity distribution, the relaxation time τ_Y and the rapidity diffusion coefficient D_Y. Rapidity spectra calculated from eq.(3) will ultimately depend on two dimensionless parameters: τ_{int}/τ_Y with the interaction time τ_{int} (the final time in the solution of eq.(3)), and $\sqrt{D_Y \tau_Y}$. The first parameter describes how close to equilibrium the system is, and the second parameter determines the width of the rapidity distribution at equilibrium. Both may be fitted from experimental rapidity distributions.

In this work, the interaction time will be taken from other sources [7]. The relaxation time τ_Y and hence, the ratio of these two times will be fitted to data. The rapidity relaxation time is related to the diffusion coefficient through the temperature of the thermal equilibrium distribution that is obtained from other origins such as hadron abundancies. The corresponding dissipation-fluctuation theorem may, however, be violated in relativistic collisions where a large amount of energy resides in the interaction. Hence, I will consider the transport coefficients as independent parameters when comparing with data.

The relaxation time determines the speed of the approach to the Gaussian stationary solution given by $\partial R_{eq}/\partial t = 0$, with the corresponding equation for the mean value

$$\frac{d}{dt}\langle Y(t)\rangle = \frac{-1}{\tau_Y}[\langle Y(t)\rangle - Y_{eq}]. \tag{4}$$

Here, τ_Y may be considered as a macroscopic quantity and - based on a suitable parametrization - determined in a fit to available data, or derived in a microscopic model. In this work I treat it on the macroscopic level. Hence, the model is positioned between the available microscopic models of BUU-type on the one hand, and macroscopic thermal models on the other hand. It maintains advantages of thermal models such as transparency, analytical treatment, and few intuitive parameters, but simultaneously goes beyond the thermalization assumption and allows to investigate nonequilibrium effects.

The diffusion coefficient D_Y is responsible for the associated broadening of the distribution functions due to collisions, particle creations and decays. For a linear FPE, the fluctuations are usually determined without detailed consideration of the microscopic structure of the system, because the stationary solution R_{eq} of the FPE should agree with the isotropic thermal equilibrium distribution R_{th}. The latter depends on a temperature T_p that is to be identified with the freeze-out temperature, and on the particle mass m_p. In the Boltzmann approximation, this thermal distribution is expressed as [2]

$$R_{th}(y_{cm}) = c_p^T m_p^2 T_p [1 + 2\chi_T(y_{cm}) + 2\chi_T(y_{cm})^2] \exp\left(\frac{-1}{\chi_T(y_{cm})}\right) \tag{5}$$

with the center-of-mass rapidity $y_{cm} = y_1 Y/2$, and

$$\chi_T(y) = \frac{T_p}{m_p \cosh(y)}. \tag{6}$$

The freezeout-temperature T_p can be approximately determined from Boltzmann fits to transverse mass spectra or hadron yields [2],[4], [8 - 11]. The constant c_p^T ensures that the distribution function in Y-space is normalized to 1 for each temperature T_p.

$$\int_{-\infty}^{+\infty} R_{th}(Y)dY = 1 \tag{7}$$

with the normalized center-of-mass rapidity $Y = 2y_{cm}/y_1$ and the projectile rapidity y_1. This yields

$$c_p^T = \frac{y_1}{2m_p^2 T_p}[\int_{-\infty}^{+\infty}(1 + 2\chi_T(y) + 2\chi_T(y)^2) \exp\left(\frac{-1}{\chi_T(y)}\right) dy]^{-1} \tag{8}$$

In thermal models of relativistic collisions [2], it is assumed that the system attains the thermal equilibrium distribution eq. (5) and subsequently expands collectively. On the basis of the present nonequilibrium-statistical approach, however, it appears that due to the short interaction time, the system does not reach the equilibrium distribution at SPS-energies, such that broad rapidity spectra could occur due to nonequilibrium properties. A possible collective expansion which might occur on top of this would require a separate treatment, it cannot be described within the FPE-framework alone.

3 Dissipation-fluctuation theorem

Equating a Gaussian approximation to the thermal distribution with the stationary solution of the FPE. I obtain the rapidity diffusion coefficient

$$D_Y^{th} = \frac{1}{2\pi\tau_Y}[c_p^T m_p^2 T_p(1+2\frac{T_p}{m_p}+2(\frac{T_p}{m_p})^2)]^{-2}\exp\left(\frac{2m_p}{T_p}\right). \qquad (9)$$

With this choice of the diffusion coefficient, thermal and stationary distribution become almost identical. For a given equilibrium distribution, the rapidity diffusion coefficient is then solely determined by the dissipative constant τ_Y. This equation is reminiscent of the well-known Einstein relation between diffusion and friction coefficient in the analysis of Brownian motion [12], $D \sim T/\tau$. Due to the temperature-dependence of the normalization constant c_p^T, it displays the expected behavior, namely, the diffusion coefficient at fixed incident rapidity y_1 rises (almost) linearly with increasing temperature of the corresponding equilibrium distribution.

In relativistic collisions, however, the above dissipation-fluctuation theorem need not be fulfilled because a large amount of energy resides in the interaction, and is used for the generation of secondary particles. Hence, in a comparison with data, values of the rapidity diffusion coefficient exceeding the ones obtained from eq. (9) may be expected, and are indeed found at SPS-energies. Consequently, I treat the transport coefficients as independent parameters at these energies.

For participant protons, δ-function initial conditions lead to superposed Gaussian solutions of the FPE with analytical expressions for mean values and variances. For small times, two separate Gaussians develop near target and projectile rapidity. They shift towards the equilibrium value and simultaneously broaden with increasing time due to collisions and particle creations. For sufficiently large ratios t/τ_Y the superposed distribution that differs from a Gaussian develops, until the distribution acquires a Gaussian shape again und then shrinks until the stationary distribution ($\Gamma_{eq} = \sqrt{8\ln 2}\sqrt{D_Y \tau_Y}$) is reached for $t \to \infty$. Due to the finite values of the interaction times (*incomplete stopping*), actual rapidity distribution functions remain significantly broader than the stationary distribution and differ from Gaussians.

Fig.1 shows distributions R (Y,t) weighted with the respective numbers of projectile and target proton participants $N_{p1,2}(b=0)$ at several selected values of t/τ_Y for the asymmetric S + Au system at SPS energies. Both the asymmetric relaxation in rapidity space and the approach to the stationary distribution at large times are clearly displayed. For a symmetric system such as Pb + Pb (fig.2), the relaxation is, of course, completely symmetric in Y. Actual rapidity density distributions for participant protons are then computed in the relativistic diffusion model as outlined in [6].

4 Comparison with experiment

Comparing these results to the data from SIS via AGS to SPS energies, I have tested the validity of the Einstein relation. At SIS-energies, the light Ni + Ni - system provides data in the range 1.06 - 1.93 A GeV [15]. With the weak-coupling value of the rapidity diffusion coefficient from eq.(9) the data are well reproduced at the lowest energy, as shown in the upper part of fig.3 (solid curve). Here, the dashed curve is the thermal equilibrium distribution with a temperature of 79 MeV (normalized to 38 protons). The thermal distribution is clearly too narrow to fit the data. In the relativistic diffusion model, the data find their natural explanation in terms of nonequilibrium contributions (incomplete stopping) as described via the solution of the FPE.The transport coefficients can be found in table 1.

At this low energy, the ratio of the interaction time for central collisions [7] to the rapidity relaxation time is about 1, indicating that the system is indeed not very far from equilibrium in rapidity space. Although this is also true at the slightly higher SIS-energies of 1.45 A GeV and 1.93 A GeV, the rapidity diffusion coefficients that allow to fit the data (fig.3) start to deviate from the values derived via the dissipation-fluctuation theorem by factors of 1.4 and 1.6, respectively, as illustrated in fig.4.

This deviation from the weak-coupling behaviour becomes more pronounced at AGS-energies. Here, the discrepancies between isotropic thermal distributions (dashed curves) [14] and data are very dramatic, fig.5. The solutions of the FPE represent the experimental results quite accurately. In case of the light Si + Al system, the nonequilibrium character is particularly evident through the dip in the rapidity spectrum, which reflects the initial situation at t=0 and appears to persist even for central collisions. The rapidity diffusion coefficients that represent the data deviate from the values obtained via the dissipation-fluctuation theorem by factors of 3.3 and 3.7 for Si + Al and Au + Au, respectively (table 1).

This violation of the dissipation-fluctuation theorem becomes even larger at SPS-energies. Since the transport coefficients depend on the mass and mass asymmetry of the system as well as on the available energy ([7],[6]), systems of the same (or at least very similar) mass and mass asymmetry should be compared when investigating the dependence on incident energy. Data for the symmetric Au + Au system are available at SIS [15] - and AGS [13] - energies, whereas Pb + Pb - data [11] are used for the comparison at SPS - energies [6]. Here, eq.(9) is violated by a factor of 9 (corresponding to a factor of 3 in FWHM).

The results for the rapidity transport coefficients are shown in fig.6. The relaxation time drops monotonically from SIS to SPS energies (upper part). For the heavy Au + Au (or Pb + Pb) system, the values of the rapidity diffusion coefficient calculated from the dissipation-fluctuation theorem do not depend much on incident energy (lower part) because the inverse proportionality to the (dropping) relaxation time is compensated by the shrinking width of the corresponding thermal equilibrium distribution in Y-space with increasing incident rapidity y_1. However, the diffusion coefficients that are required to fit the data are seen to rise from SIS-energies, where the value from eq.(9) gives a reasonable representation of the data at 1.77 A GeV/c (E_{beam} = 1.06 A GeV), via AGS- to SPS- energies, where clear violations of eq.(9) are observed.

This conclusion is not affected by the present uncertainties in the interaction times that enter the model calculation, because rescaled values of the interaction times (and correspondingly, of the rapidity relaxation time, keeping the ratio constant) would modify the diffusion coefficients according to eq.(9), but maintain the variances. Hence, the violation of the dissipation-fluctuation theorem in Y-space with increasing energy occurs independently of a detailed description of the dynamical behaviour, it is a consequence of the nonequilibrium-statistical properties of the system.

In transport descriptions of heavy-ion collisions at non-relativistic energies, one has tried to pin down a violation of the Einstein relations using various macroscopic variables (distance between the two center-of-mass, mass asymmetry, deformation). In spite of detailed theories, no compelling evidence was found for such a violation [16] when comparing with data. For the rapidity variable at relativistic energies, however, the evidence is quite straightforward [17].

It would be interesting to investigate in more detail the origin of this effect in relativistic collisions. Whereas it could be due to the strong-coupling behaviour that leads to an increase of the diffusion coefficient above the values that I have obtained from eq.(9), a rapid collective expansion of the system (before thermal equilibrium is attained) also leads to broader distributions and hence, could cause an artificial enhancement in the diffusion coefficients that are used to represent the data. The enhancement might eventually also be due to non-equilibrium quark-gluon plasma formation prior to hadronization. Developing unambiguous tests to discriminate these possibilities is very challenging.

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Table 1

Relaxation times τ_Y and rapidity diffusion coefficients D_Y at SIS-, AGS- and SPS-energies. The incident momentum per particle is p_L, the temperature of the thermal equilibrium distribution T_p. The theoretical value of the diffusion coefficient corresponding to the Einstein relation is D_Y^{th}, whereas D_Y is the fit value.

System	p_L (GeV)	y_1	τ_Y (fm)	τ_{int}/τ_Y	T_p (MeV)	D_Y^{th} (10^{-4}fm^{-1})	D_Y (10^{-4}fm^{-1})
Ni + Ni	1.77	1.388	360	1.03	79	4.7	4.7
	2.20	1.586	310	1.02	84	4.4	6.2
	2.71	1.782	290	0.95	92	4.1	6.5
Si + Al	14.6	3.44	60	0.80	140	7.8	26.0
Au + Au	1.77	1.39	1100	1.14	80	1.6	1.6
Au + Au	11.4	3.19	370	1.08	130	1.4	5.1
Pb + Pb	157.7	5.83	134	0.81	160	1.4	12.4
S + S	200	6.06	25	0.60	140	7.4	88.7
S + Au	200	6.06	72	0.84	175	2.1	25.5

Fig.1. Rapidity relaxation of participant protons in 200 A GeV/c S + Au. Selected analytical solutions of the Fokker-Planck equation for various values of time t/τ_Y weighted with the respective numbers of projectile and target participants for central collisions illustrate the asymmetric equilibration. The dashed curve is the Gaussian stationary distribution (centered at $y_{eq} = 2.63$, $Y_{eq} = -0.13$).

Fig.2. Rapidity relaxation of participant protons in 158 A GeV/c Pb + Pb. Selected analytical solutions for various values of time t/τ_Y as in fig.1 are shown. The stationary distribution (dashed) is centered at midrapidity ($y_{eq} = 2.91, Y_{eq} = 0$).

Fig.3. Rapidity relaxation of participant protons in central relativistic Ni + Ni- collisions at SIS- energies. FOPI data are reflected about midrapidity. The dashed curves are thermal equilibrium distributions, solid curves are nonequilibrium results of the Relativistic Diffusion Model. At the lowest energy, the rapidity diffusion coefficient is obtained from the dissipation-fluctuation theorem, at the higher energies enhanced values are used to fit the data.

Fig.4. Dependence of the rapidity diffusion coefficients on incident momentum per particle in Ni + Ni reactions at SIS energies. The fit values deviate from the results of the dissipation-fluctuation theorem at the higher momenta. The lines are drawn to guide the eye.

Fig.5. Rapidity relaxation of participant protons at AGS energies (cf. table 1). The Au + Au data and part of the Si + Al data have been reflected about midrapidity. Isotropic thermal equilibrium dis-

tributions (dashed curves) are too narrow to fit the data. Agreement is achieved through the nonequilibrium solutions in the RDM, solid curves. The fit values of the diffusion coefficients exceed the ones obtained from the dissipation-fluctuation theorem.

Fig.6. Dependence of the rapidity relaxation time and the rapidity diffusion coefficient on the incident momentum per particle in the energy-range from SIS via AGS to SPS. The values of the diffusion coefficients as derived from fits to Au + Au and Pb + Pb data are significantly larger than results from the dissipation-fluctuation theorem at the higher energies. Lines are drawn to guide the eye.

Fig.7. Calculated rapidity density spectra of participant protons in central S+Au, S+S and Pb+Pb- collisions at SPS- energies in comparison with NA 35 and NA 49 data (black squares; data for symmetric systems reflected at $y_{eq} = y_1/2$). The projectile rapidities are $y_1 = 6.06$ for S, and $y_1 = 5.83$ for Pb. For Pb+Pb the thermal equilibrium distribution (T=160 MeV) is also shown; its maximum is at dn/dy = 165 (dashed). The data are incompatible with isotropic thermal distributions. The fit values of the rapidity diffusion coefficients are significantly larger than the ones caculated from the weak-coupling dissipation-fluctuation theorem and hence, the Einstein relation is violated.

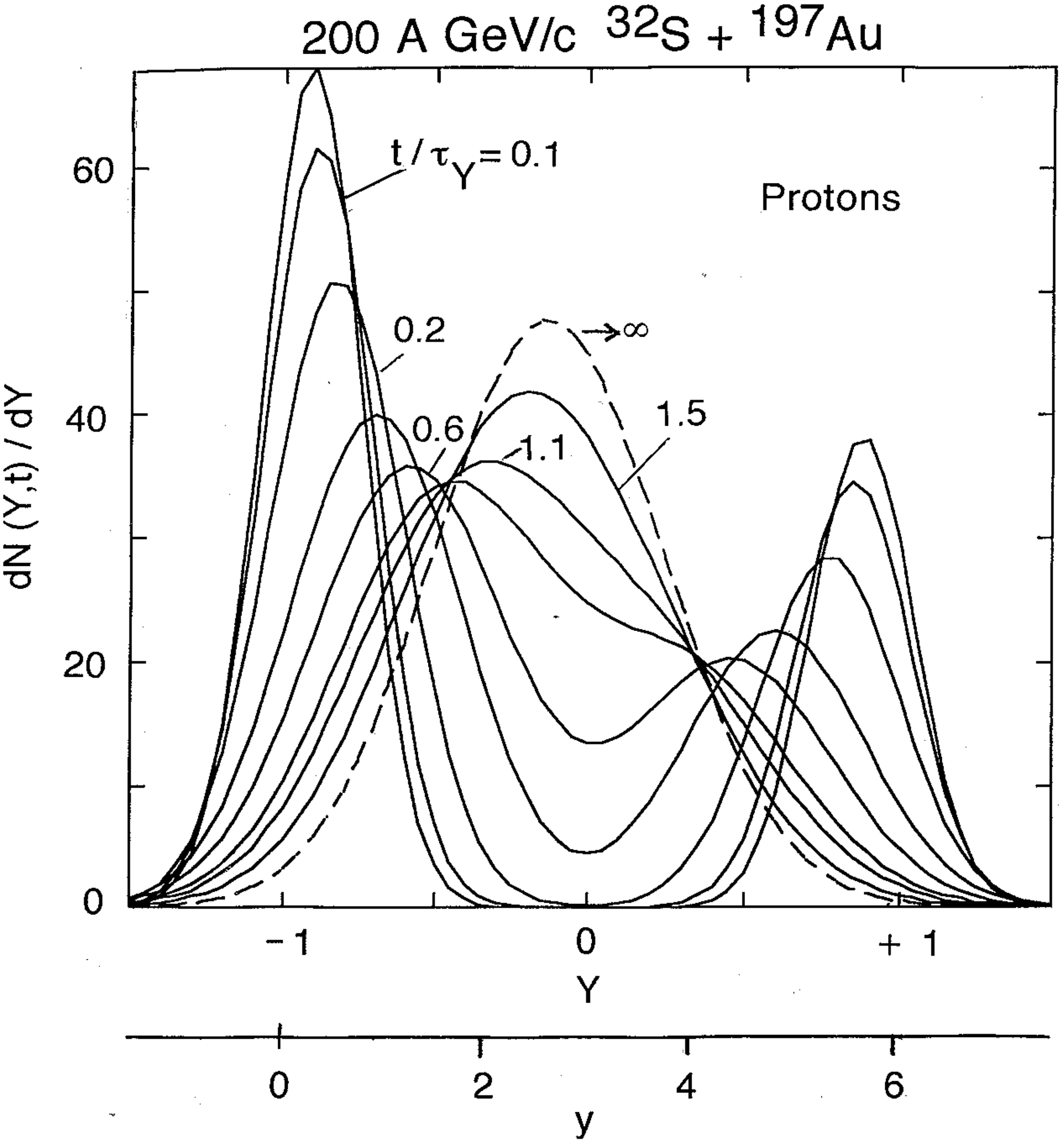

200 A GeV/c $^{32}S + {}^{197}Au$
$t/\tau_Y = 0.1$
Protons
0.2
→∞
0.6
1.1
1.5
dN (Y,t) / dY
60
40
20
0
−1
0
+ 1
Y
0
2
4
6
y

Fig.1.

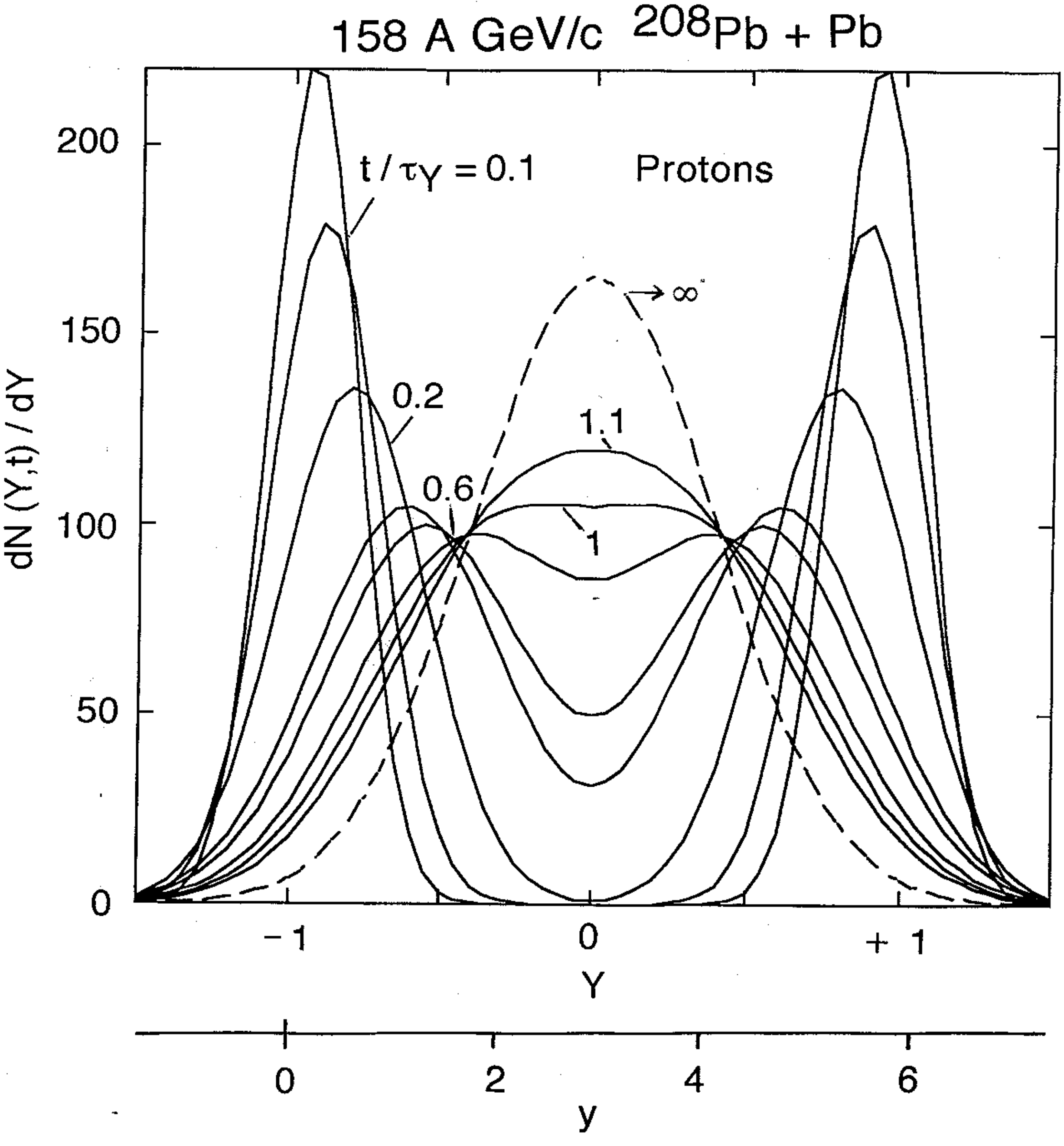

158 A GeV/c ^{208}Pb + Pb
200
150
100
50
0
$t / \tau_Y = 0.1$
Protons
$\rightarrow \infty$
0.2
1.1
0.6
1
dN (Y,t) / dY
− 1
0
+ 1
Y
0
2
4
6
y

Fig.2.

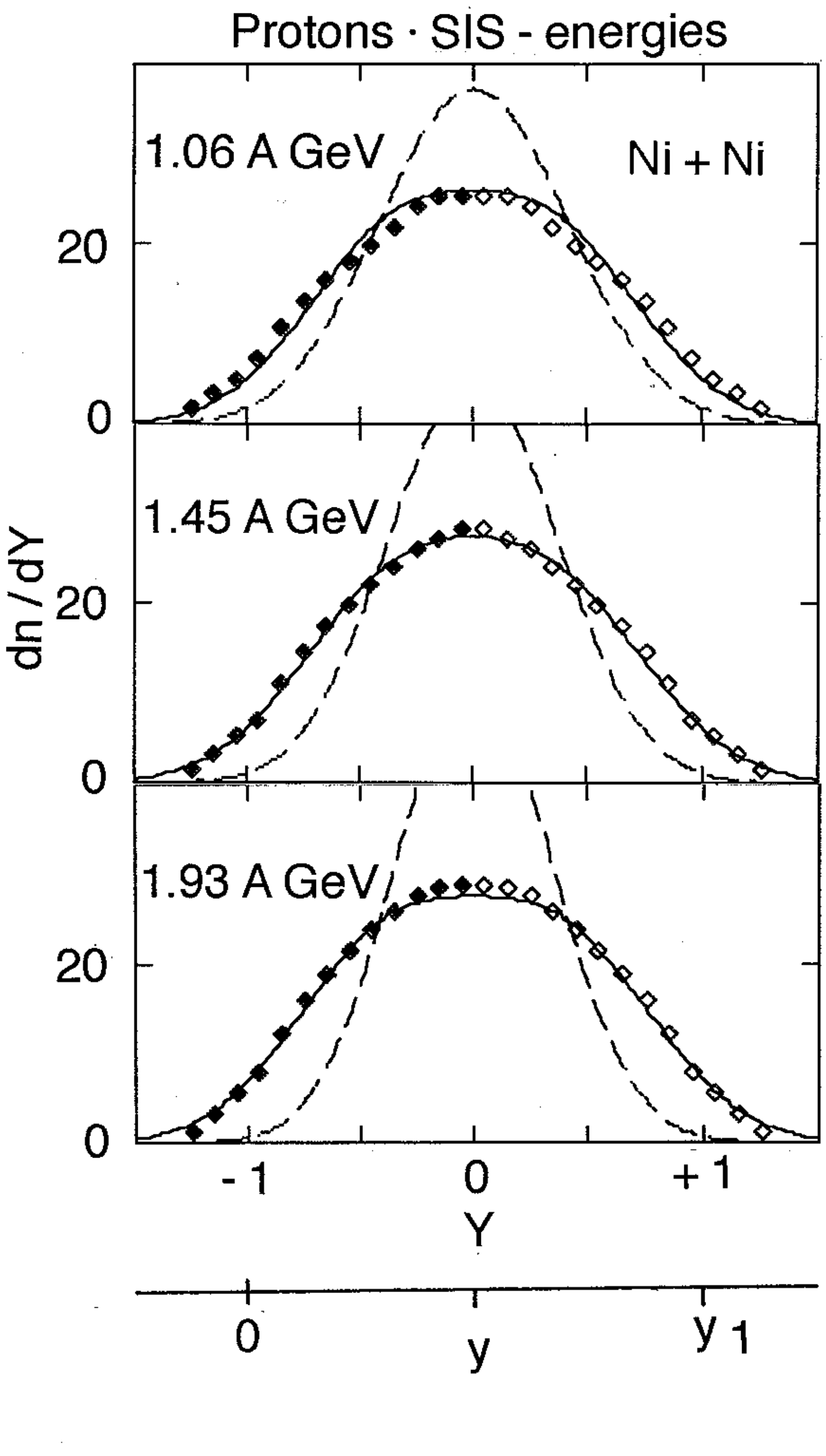
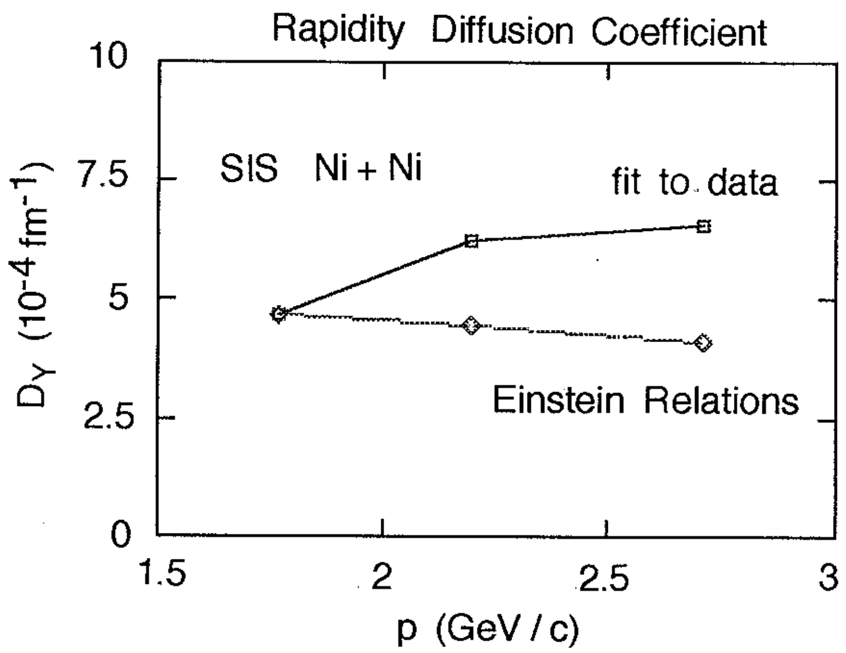

Protons · SIS - energies
1.06 A GeV
Ni + Ni
1.45 A GeV
1.93 A GeV
dn / dY
20
0
- 1
0
+ 1
Y
0
y
y 1

FIG. 1.



Rapidity Diffusion Coefficient
10
7.5
5
2.5
0
1.5
2
2.5
3
SIS Ni + Ni
fit to data
Einstein Relations
D_Y (10^{-4} fm^{-1})
p (GeV / c)

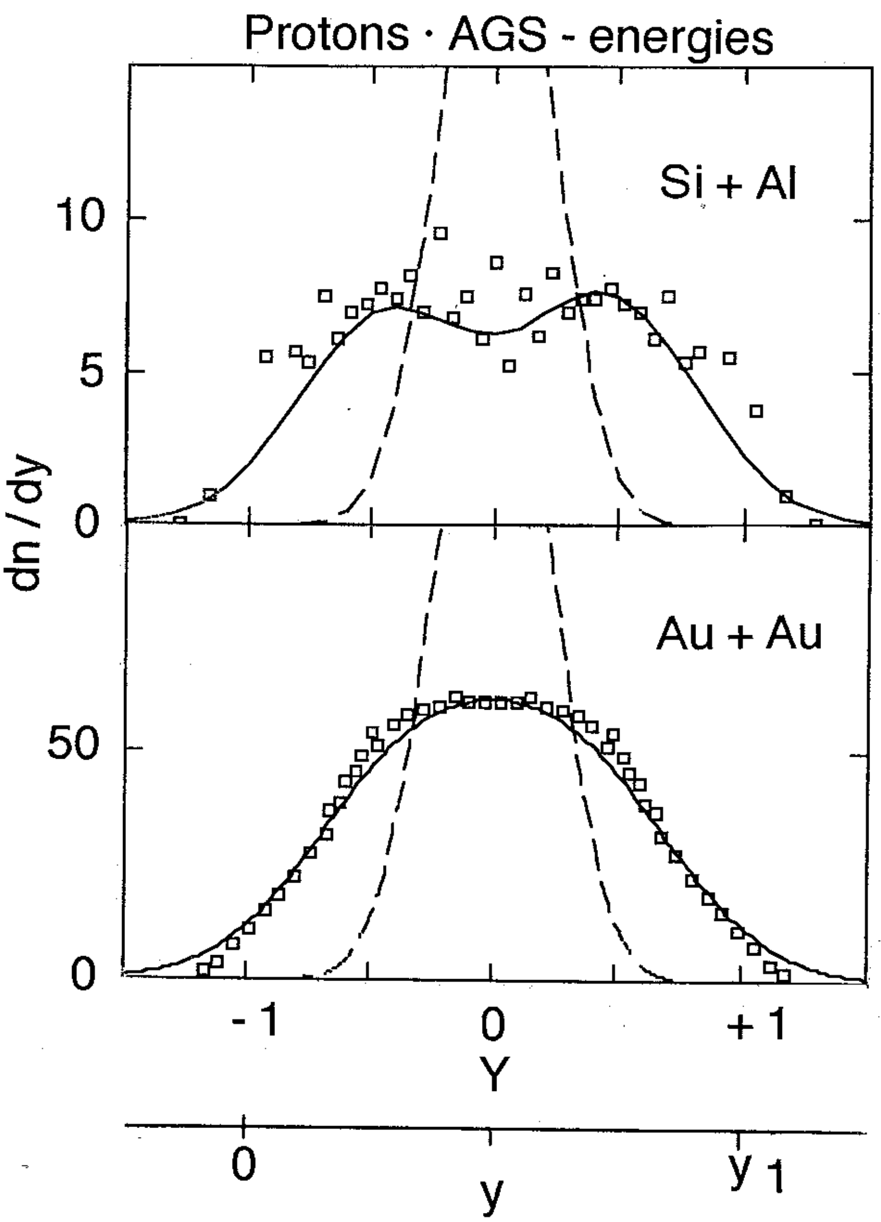
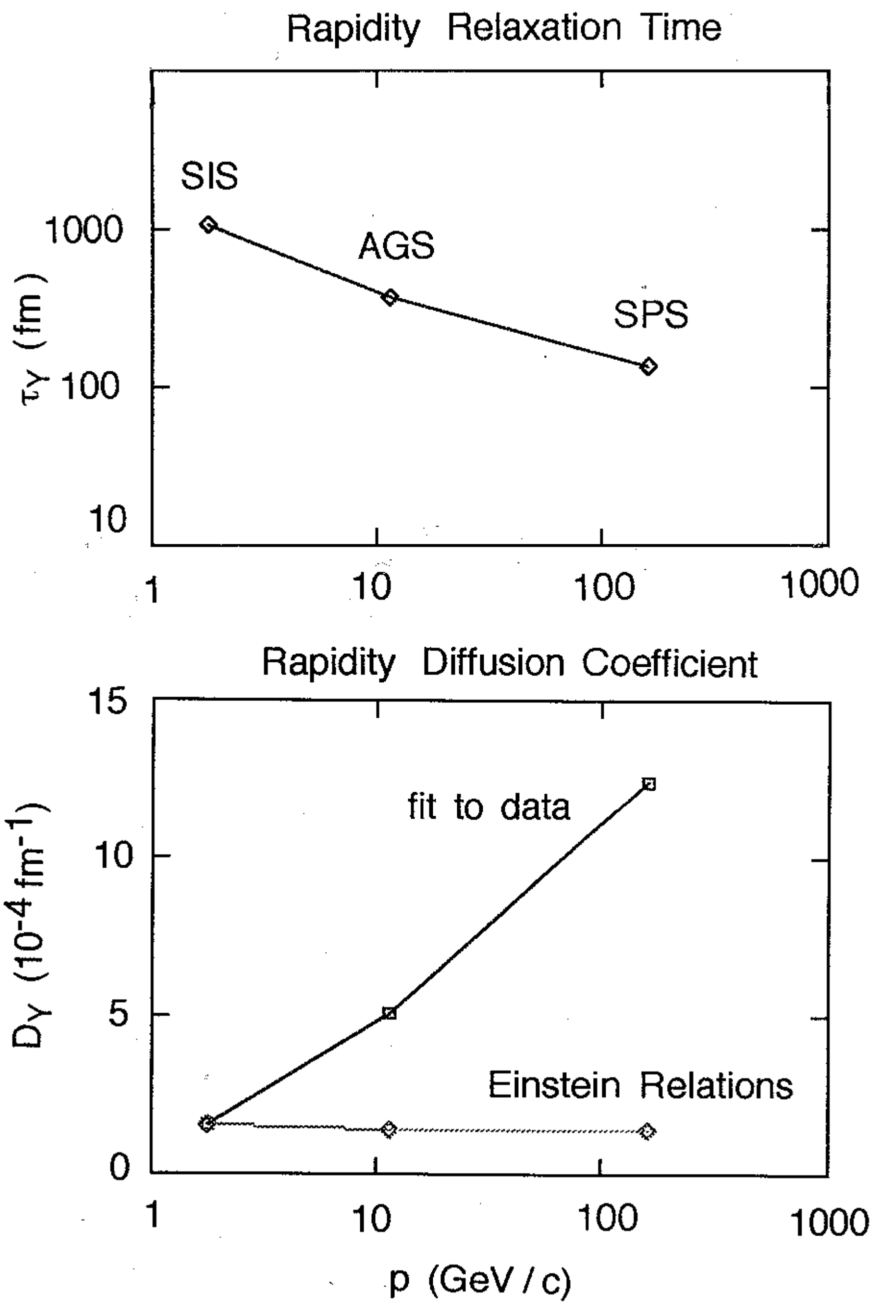


FIG. 3.


Rapidity Relaxation Time
SIS
AGS
SPS
τ_Y (fm)
1000
100
10
1
10
100
1000
Rapidity Diffusion Coefficient
fit to data
Einstein Relations
D_Y (10^{-4} fm^{-1})
15
10
5
0
1
10
100
1000
p (GeV / c)

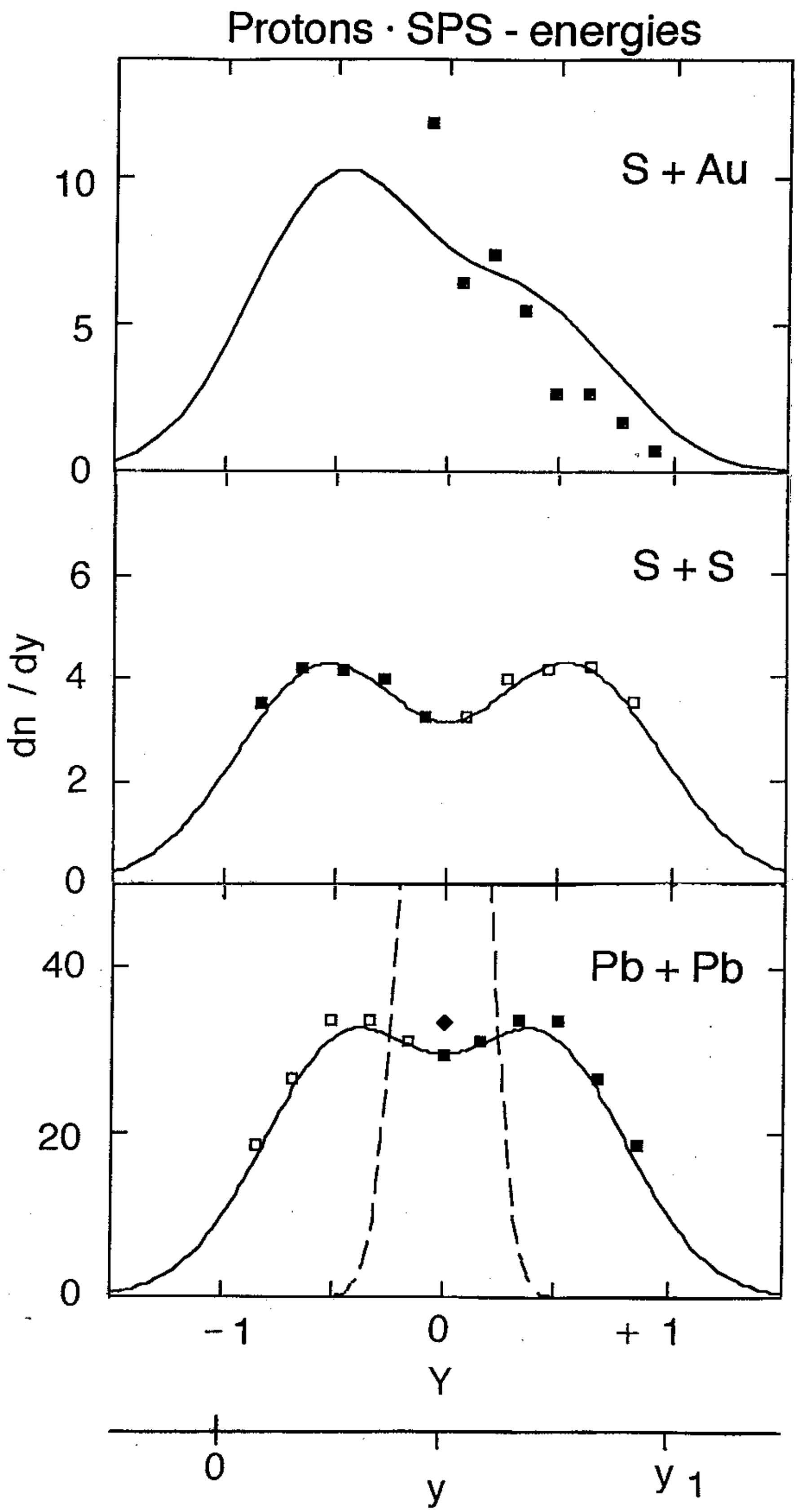

Protons · SPS - energies
S + Au
10
5
0
S + S
6
4
2
0
dn / dy
Pb + Pb
40
20
0
− 1
0
+ 1
Y
0
y
y 1

Fig.3.