Strong-coupling diffusion in relativistic systems

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Different from the early universe, heavy-ion collisions at very high energies do not reach statistical equilibrium, although thermal models explain many of their features. To account for nonequilibrium strong-coupling effects, a Fokker-Planck equation with time-dependent diffusion coefficient is proposed. A schematic model for rapidity distributions of participant baryons is set up and solved analytically. The evolution from SIS via AGS and SPS to RHIC energies is discussed. Strong-coupling diffusion produces double-peaked spectra in central collisions at the higher SPS momentum of 158 A GeV/c and beyond.

1 Introduction

Colliding many-body systems such as heavy ions are particularly suited for the study of diffusion processes at relativistic energies. Once the available energy exceeds the rest mass substantially (at SPS, eightfold; at RHIC and LHC, by factors of 105 and 2940, respectively), strong-coupling effects resulting in copious quark-gluon production with subsequent hadronization become important.

In this note, I formulate a schematic model that accounts for these effects through a time-dependent rapidity diffusion coefficient. It generally exceeds the weak-coupling value, and approaches it only for large times beyond hadronization. With the weak-coupling value, good descriptions of the nonequilibrium features in case of SIS proton rapidity spectra at the low incident momentum of 1.77 A GeV/c have been obtained. In contrast, the enhanced values needed to interpret the data at higher energies point towards the importance of strong-coupling effects.

I solve the corresponding Fokker-Planck equation (FPE) in rapidity space analytically for the case of a time-dependent diffusion coefficient. The solutions represent SPS-data for 158 A GeV/c Pb+Pb very well. With the dependence of the rapidity relaxation coefficients on the available relativistic energy, results at 40 A GeV/c are obtained. Extrapolating to higher energies, an estimate of the expected proton rapidity spectra at RHIC is also given.

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2 Relativistic diffusion model for strong coupling

In order to describe the gradual approach of the participant baryons in the colliding system to statistical equilibrium, I introduce the dimensionless time-like rapidity relaxation variable $\theta_Y = t/\tau_Y$ with the rapidity relaxation time τ_Y. The variable θ_Y determines how close to equilibrium the rapidity distribution is at a time t. For t equal to the value of the interaction time τ_{int} (the final time in the integration of the transport equation), the relaxation constant becomes $\Theta_Y^{int} = \tau_{int}/\tau_Y$. The width of the rapidity distribution is governed by the dimensionless rapidity relaxation coefficient

$$\Delta_Y = \sqrt{D_Y \tau_Y} \tag{1}$$

with the diffusion coefficient D_Y of that governs the broadening due to incoherent interactions and particle productions. The rapidity relaxation time and the diffusion coefficient are related through the weak-coupling dissipation-fluctuation theorem (Einstein relation). It is due to the fact that the stationary solution of the transport equation in rapidity space has to agree with the thermal equilibrium distribution. Hence, the equilibrium distribution defines the diffusion coefficient in weak coupling, and the corresponding dimensionless rapidity relaxation coefficient becomes

$$\Delta_Y^w = \Big\{\frac{1}{2\pi}\Big[C_p^T\Big(1+2\frac{T_p}{m_p}+2(\frac{T_p}{m_p})^2\Big)\Big]^{-2}\exp\left(\frac{2m_p}{T_p}\right)\Big\}^{\frac{1}{2}} \tag{2}$$

with the temperature T_p of the corresponding equilibrium distribution that is approached for $\theta \to \infty$, the proton mass m_p and the temperature-dependent constant C_p^T. Rapidity spectra calulated using this theoretical rapidity width coefficient are in good agreement with low-energy SIS data. However, data at AGS-energies and beyond disagree with the theoretical result due to strong-coupling effects. To account for strong coupling, I introduce a time-dependent diffusion coefficient through a relaxation ansatz

$$\frac{\partial D_Y(t)}{\partial t} = -\frac{1}{\tau_s}\Big[D_Y(t) - D_Y^w\Big]. \tag{3}$$

According to the relaxation ansatz, the diffusion coefficient for weak coupling D_Y^w is approached for times $t \gg \tau_s$ larger than the time τ_s that is characteristic for strong coupling - when all secondary particles have been created. For short times in the initial phase of the collision before and during particle production, the strong-coupling diffusion coefficient D_Y^s dominates and enhances the diffusion in Y-space beyond the weak-coupling value. This enhancement rises strongly with incident energy. It is decisive for a proper representation of the available data for relativistic heavy-ion collisions at and beyond SPS energies.

The rapidity transport equation is a Fokker-Planck equation (FPE) in Y-space. To account for the strong-coupling case, however, a time-dependent diffusion coefficient has to be introduced, which complicates the previously simple analytical solution. Rewriting the FPE in terms of the dimensionless variables θ, Δ yields

$$\frac{\partial R(Y,\theta)}{\partial\theta} = \frac{\partial}{\partial Y}\Big[(Y - Y_{eq})R(Y,\theta)\Big] + \Delta_Y^2(\theta)\frac{\partial^2}{\partial Y^2}R(Y,\theta). \tag{4}$$

The FPE in rapidity space has superposed Gaussian solutions. I solve the differential equations for the first and second moments analytically. The solution for the variances can then be written as a sum of weak- and strong-coupling contributions.

In the limit of $\tau_s \to 0$ the fluctuations due to strong coupling vanish, and the remaining weak-coupling result for the variance of the rapidity distribution function attains the familiar form

$$\sigma_Y^2(\theta) \to \Delta_Y^2\Big[1 - exp(-2\theta_Y)\Big] \quad for \quad \tau_s \to 0. \tag{5}$$

3 Comparison with data and predictions

With the time-dependent solutions of the FPE that are defined in terms of the mean values and the variances derived in the last section, I obtain rapidity distributions that may be compared to data. At low SIS-momentum of 1.77 A GeV/c, the strong-coupling features are not yet pronounced and the analytical weak-coupling expression yields good results. At AGS-energies and beyond, the strong-coupling behavior is no longer negligeable.

For 158 A GeV/c Pb+Pb at the SPS, the proton rapidity spectrum calculated with the strong-coupling variance shows a pronounced dip at midrapidity, in agreement with the data. This indicates that the system of participant baryons is far from equilibrium and hence, thermal models are not adequate for the participants, although they may represent produced particles quite well.

4 Conclusion

Whereas the equilibration of colliding many-particle systems at comparatively low relativistic (SIS) energies can be modelled in a weak-coupling diffusion framework in rapidity space, at SPS-energies and beyond a strong-coupling treatment is clearly required. I have solved the corresponding Fokker-Planck equation with a time-dependent rapidity diffusion coefficient analytically.

The comparison with participant baryon rapidity spectra at the higher SPS energies confirms that the system is far from equilibrium, and dominated by strong-coupling effects. Only for large times when all secondary particles have been produced from the available energy does the rapidity diffusion coefficient approach the weak-coupling limit, which can be obtained analytically from the equilibrium distribution with given temperature T.

Results based on an extrapolation of the strong-coupling rapidity relaxation coefficients as functions of the available relativistic energy show that at the lower SPS energy of 40 A GeV, the system is still far from equilibrium in central collisions, but the rapidity distribution of baryons for central collisions is flat and hence, the nonequilibrium situation is not directly evident.

At RHIC energies, however, the rapidity distribution of baryons (without secondary ones from hyperon decays) is expected to show two clearly separated peaks which will indicate that the system of baryons - no matter whether it passes through a deconfined phase or not - does not reach statistical equilibrium.

Figure caption

Participant proton rapidity spectra for central collisions of heavy systems at SPS and RHIC energies in the RDM for strong coupling. Superposed nonequilibrium distributions produce the broad plateau at 40 GeV/c per nucleon. Here the data are not yet available. At 158 GeV/c per nucleon the nonequilibrium features are clearly present in the data. At RHIC-energies, the nonequilibrium shape of the proton rapidity spectrum (solid curve) is well developed. The dash-dotted line is a nonequilibrium calculation with an enhanced value of the diffusion coefficient (corresponding to a width coefficient Δ_Y=0.49). Dashed lines are isotropic thermal equilibrium distributions (calculated here with a common temperature of 160 MeV).