Fixing D7-Brane Positions by F-theory Fluxes

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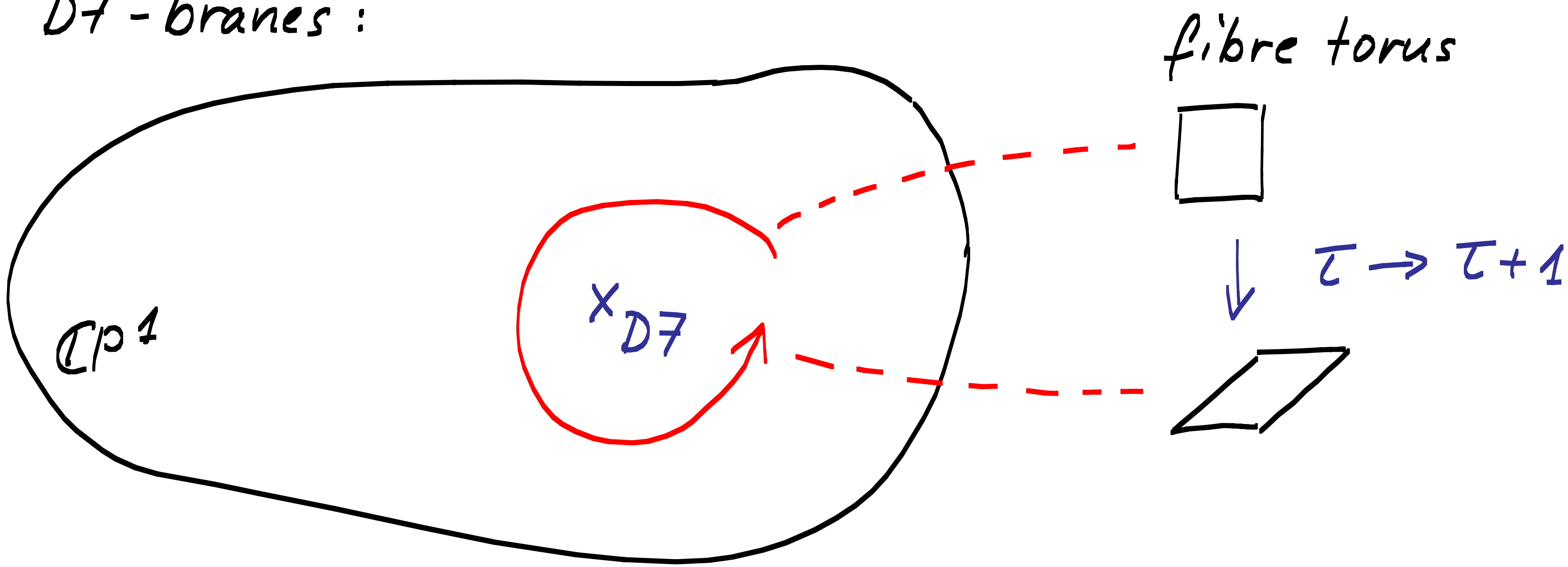
Motivation

- Model building in type IIB with D7-branes / F-theory
- Long-term goal: Concrete, global models with methods of algebraic geometry (this may be most straightforward in F-theory)
- here: Mainly K3×K3 + some ideas beyond ...

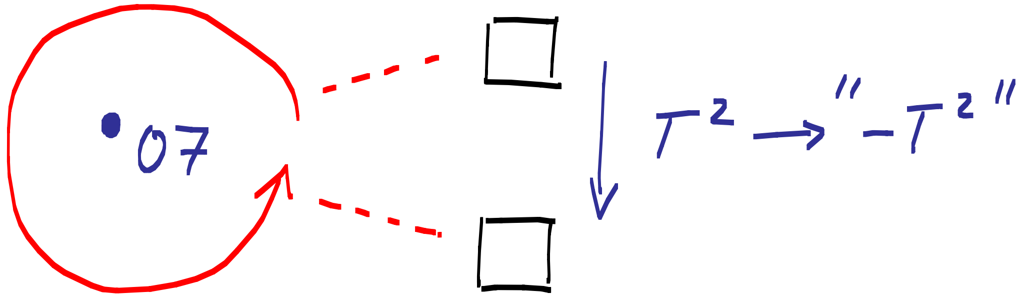
M-theory on $\mathbb{R}^7 \times K3$

- Think of K3 as T^2-fibration over $\mathbb{CP}^1$
- $\mathrm{Vol}(T^2) \to 0 \quad \Rightarrow \quad$ type IIB on $\mathbb{R}^8 \times \mathbb{CP}^1$

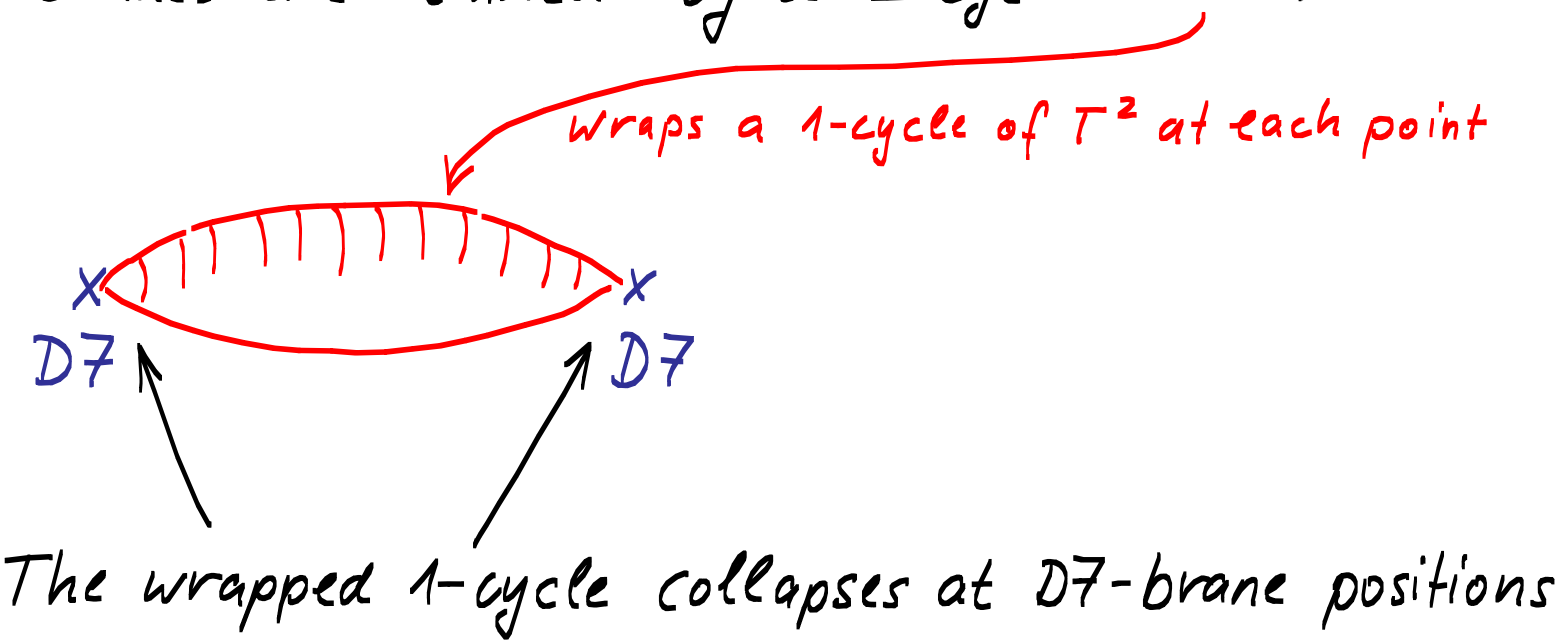
  (first type IIA in 7d,
  then type IIB in 8d via T-duality)
- D7-branes:



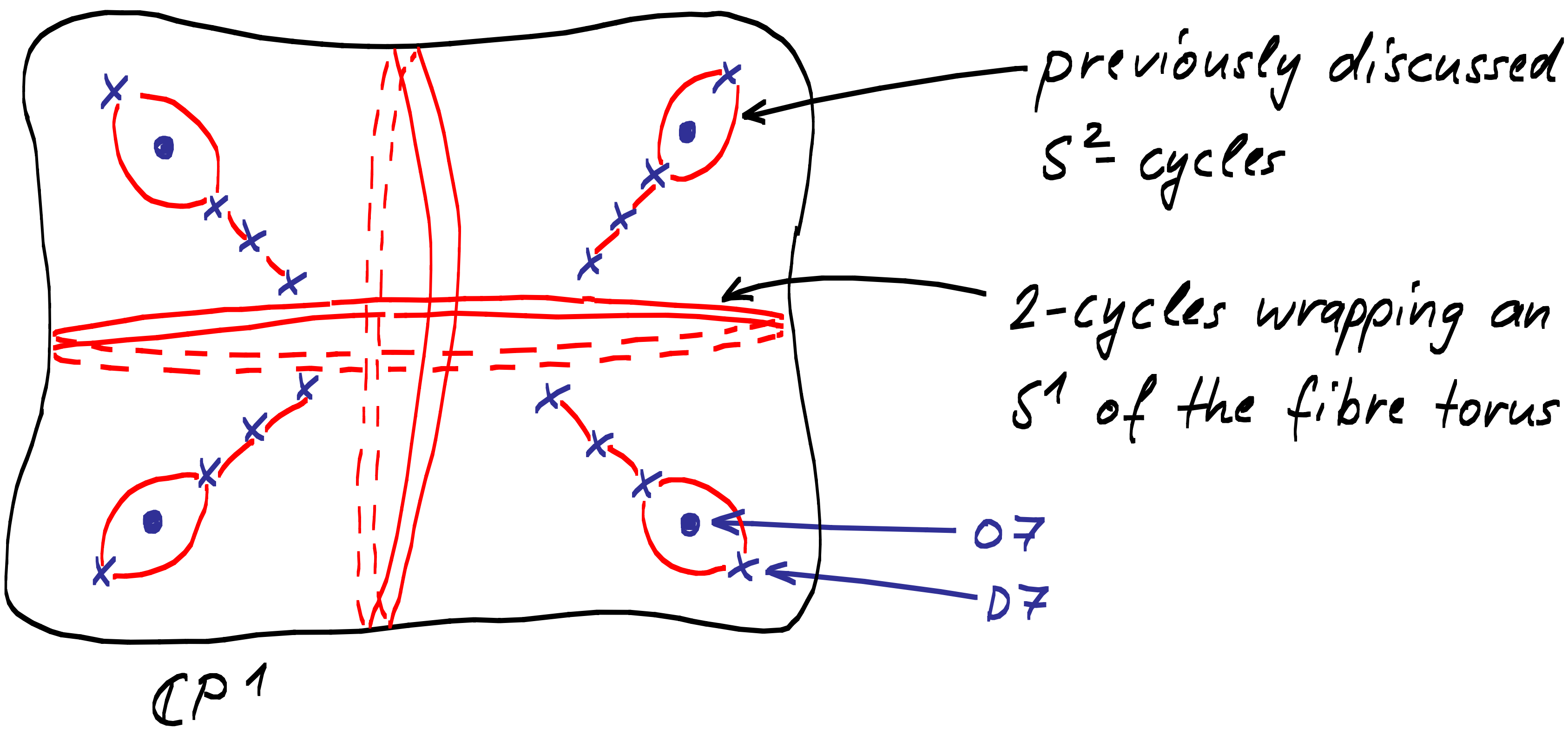
- Analogously for an O7-plane (with 4 D7-branes):



- Two D7-branes are "linked" by a 2-cycle $\sim S^2$:



K3 = $\mathbb{C}P^1$ with 4 O7-planes & 16 D7-branes:



These 2-cycles (+ base & fibre) can be explicitly identified with $H_2(K3, \mathbb{Z})$ with its intersection metric

(→ A. Braun, A.H., H. Triendl, 01/08)

Now think of $H_2(K3)$ as a 22-dimensional vector space with the intersection metric (of signature $(3,19)$).

$$\Omega = \omega_1 + i\omega_2 \quad ; \quad j = (\mathrm{Vol.}) \cdot \omega_3$$

3 orthonormal vectors

$\Rightarrow$ define a 3-plane in $H^2(K3)$

(The size of a cycle is determined by projection onto this plane.)

4-form flux on $K3 \times \tilde{K3}$:

$$G_4 = \sum_{IJ} \eta_I \wedge \tilde{\eta}_J \, G^{IJ}$$

integral bases

$\Rightarrow$ G_4 has natural interpretation as map $G: H_2(\tilde{K3}) \to H_2(K3)$

Flux potential:

(calculated following Haack, Louis '01)

$$V = -\frac{1}{2\,\mathrm{Vol}^3}\cdot\left(\sum_i\left|\tilde{\mathbb{P}}[G^A\omega_i]\right|^2 + \sum_j\left|\mathbb{P}[G\tilde{\omega}_j]\right|^2\right)$$

projectors on space orthogonal to 3-plane

Note: V has manifest $SO(3)\times SO(3)$ symmetry.

Flux-stabilized minima:

$V=0 \iff G$ (and its adjoint G^A) map the 3-plane of one K3 on the 3-plane of the other K3.

Implications:

- Flux stabilizes geometry with $V=0$

  $\Leftrightarrow$ $G^A G$ ($\& \, G G^A$) has 3 positive-norm eigenvectors with eigenvalues ≥ 0

  (Note: This is a non-trivial restriction because of signature $(3,19)$)

- These eigenvectors determine the two planes

- Unstabilized moduli remain if there exist negative-norm eigenvectors with degenerate eigenvalues.
  (The plane can then be rotated in this direction.)

To make things concrete, recall the intersection-metric in in the previously discussed 2-cycle basis:

Base-fibre block

$$
\begin{pmatrix}
\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} & & & & & & \\
& \begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix} & & & & & \\
& & \begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix} & & & & \\
& & & \boxed{D_4} & & & \\
& & & & \boxed{D_4} & & \\
& & & & & \boxed{D_4} & \\
& & & & & & \boxed{D_4}
\end{pmatrix}
$$

cycles wrapping base

root lattice of SO_8^4

- let us now stabilize a geometry with gauge symmetry $SO_8{}^3 \times SO_4 \times SU_2$, i.e.
  - 3 O-planes carry 4 D-branes
  - 1 O-plane carries 2 D-branes
  - 2 D-branes are separated from O-planes but together

Procedure:

- Identify cycles which we want to shrink (No flux goes there!)
- Identify base-fibre block (No flux goes there. However, $j \sim \omega_3$ will be in this block)
- Identify an integral basis of the remaining lattice (it has signature $(2,3)$ and contains the cycle governing the "distance between" SO_4 and SU_2)

- An appropriate flux matrix is:

base-fibre block

$$\begin{pmatrix} \boxed{0} & & & & & & \\ & 1 & 1 & & & & \\ & 1 & 1 & & & & \\ & & & 1 & 1 & 1 & \\ & & & 1 & 3 & 1 & \\ & & & 1 & 1 & 2 & \\ & & & & & & \boxed{0} \end{pmatrix}$$

(2,3) block with two 2-planes, fully stabilized

$SO_8^4 \times SO_4 \times SU_2$ block

<u>Note:</u> Technically, we work by "trial and error" checking integral flux matrices in the selected block for

1) $\operatorname{tr} G^A G = 48$ (tadpole; very restrictive!)

2) existence & positivity of eigenvalues of $G^A G$.

<u>Comment:</u> A related analysis is contained in Aspinwall, Kallosh, '05

<u>However:</u>

- They restrict themselves to planes containing a maximal-dimension sublattice (⇒ attractive K3 surfaces)
- This implies a much smaller set of allowed fluxes
- They do not relate to type IIB geometry and the determination of different gauge groups

Outlook:

① $K3 \times K3 / \mathbb{Z}_2$ (with $\mathbb{Z}_2$ acting freely on at least one of the $K3$'s)

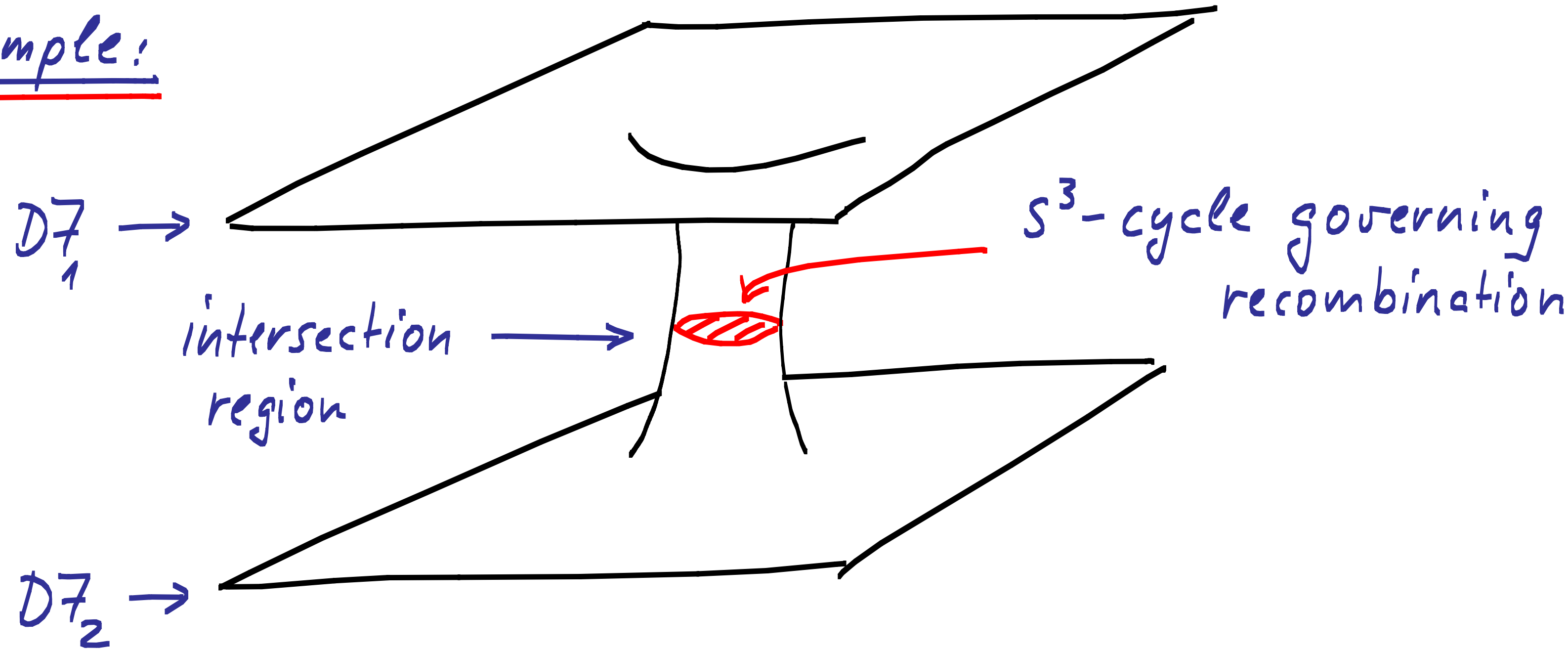
$\Rightarrow$ $N=1$ SUSY in 4d ; closer to proper CY_4

but: no intersecting branes yet

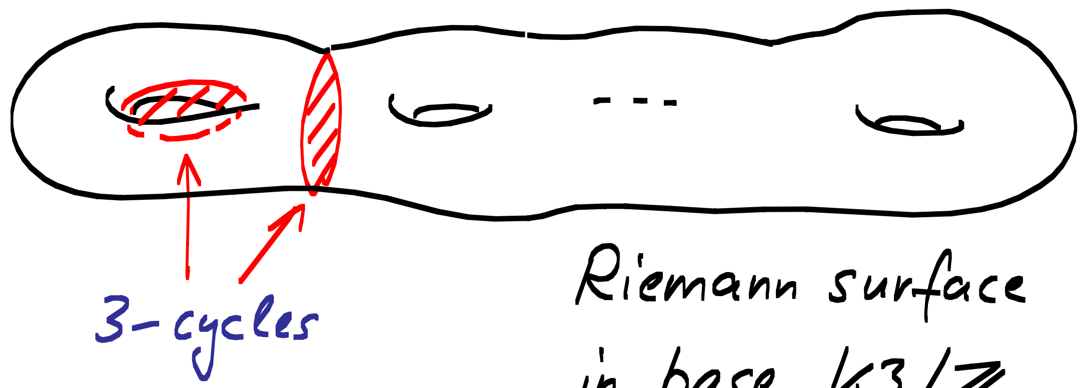
② In the case of M-theory on CY_3, we have determined a large portion of the 3-cycles (and their intersection structure) governing D7-brane motion.

We understand (in principle) brane recombination and brane & O-plane motion via F-theory cycles:

An example:



"Total" D7-brane:



Riemann surface in base $K3/\mathbb{Z}_2$ of type IIB

But: This is only 6d, not yet 4d ...