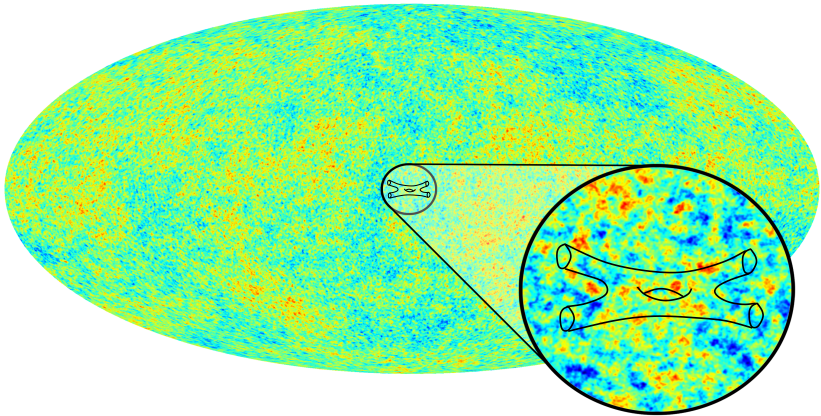


# Recent Progress in String-Theoretic Models of Cosmological Inflation



Background Image: Planck Collaboration and ESA

# Recent Progress in String-Theoretic Models of Cosmological Inflation

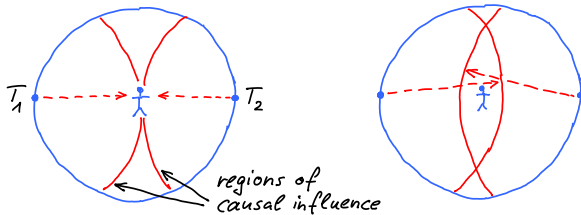
Arthur Hebecker (Heidelberg)

## Outline

- Preliminaries: The need for inflation
- Inflation in field theory
- Why look for inflation in string theory
- The (flux-) landscape, eternal inflation and the multiverse
- Problems with large-field inflation in string theory
- Axion monodromy - early models and recent progress

## The need for inflation

- Inflation has become the dominant paradigm for early cosmology
- One of the reasons is the 'Horizon Problem'
- In short, the problem is that:  
We observe homogeneity between regions which have never been in causal contact with each other



- Crucial: The **extra** time between zero and decoupling is very small (cf. right-hand picture)

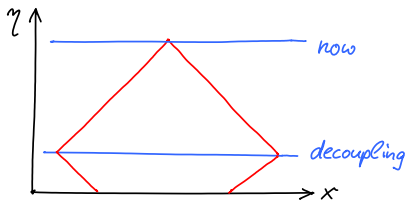
## The need for inflation (continued)

- To be more precise, start from the Friedman-Robertson-Walker metric:

$$ds^2 = dt^2 - a^2(t)d\vec{x}^2 \quad , \quad a(t) \sim t^{2/3}$$

- Change coordinates according to  $d\eta = dt/a(t)$  (conformal time):

$$ds^2 = a^2(\eta) [d\eta^2 - d\vec{x}^2]$$



- The plot makes the 'shortness' of the time before  $\eta_{decoupling}$  manifest

## Comment

- Of course, at  $t = 0$  (or at  $\eta = 0$ ), the whole universe is just a point
- Thus, one could say that at this 'big-bang singularity' everything is in causal contact anyway
- But to make this quantitative, one needs to be able to calculate at Planck-scale energy-densities
- Such attempts have indeed been made, but they depend on even conceptually unknown physics

## Inflation solves the horizon problem

- Inflation introduces an early period in cosmology dominated by  $\Lambda_{\text{cosm.}} = V(\varphi)$
- During this period, the universe expands exponentially:  $a(t) \sim e^{Ht}$ , where  $H \sim \sqrt{\Lambda}/M_P$
- This expansion is so fast, that even tiny regions (where everything is in causal contact) are blown up to sizes much bigger than the whole observable universe
- To check this quantitatively, just redo the previous plot with  $a \sim e^{Ht}$

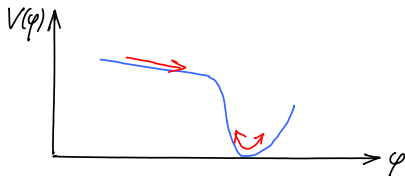
Starobinsky '80  
Guth '81  
Linde '82

## Inflation in field theory

- The simplest relevant action is (from now on  $M_P = 1$ )

$$S = \int d^4x \sqrt{g} \left[ \frac{1}{2} R[g_{\mu\nu}] + \frac{1}{2} (\partial\varphi)^2 - V(\varphi) \right]$$

- We can realise inflation if  $V(\varphi)$  has a sufficiently flat region

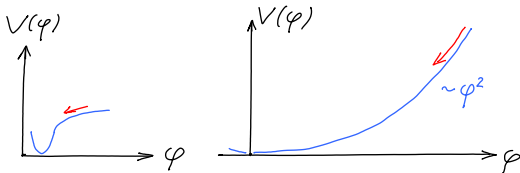


(More quantitatively, we need  $V'/V \ll 1$  and  $V''/V \ll 1$ )

- In the end,  $\varphi$  oscillates and decays to SM particles  
(‘reheating’  $\equiv$  ‘big bang’)

## Inflation in field theory (continued)

- If we allow ourselves to draw  $V(\varphi)$  'by hand', we can make some part of it **very flat**
- In this case,  $\varphi$  rolls **very slowly**, i.e. we get enough inflation (number of e-foldings) with  $\Delta\varphi \ll 1$
- Such models are called '**small field**' models



- Alternatively, one can use 'generic' potentials (e.g.  $V(\varphi) \sim \varphi^2$ )
- In such **large field** models, one needs  $\Delta\varphi \gg 1$   
(We will see that this is a challenge in string theory)



## Why look for inflation in string theory?

- Different types of questions have different sensitivity to the **UV-completion / quantum gravity effects / string theory**
- I want to argue that inflation is **very** sensitive to the UV
- **Key point:** In field-theory + quantum gravity we generically have higher-dimension operators  $\sim \varphi^6/M_P^2 \equiv \varphi^6$  etc.
- Such effects may endanger the extreme flatness at  $\varphi \ll 1$  or be completely fatal at  $\varphi \gg 1$

## A small warning / disclaimer:

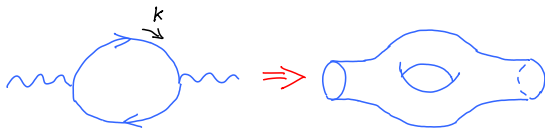
- It is not impossible to ensure flatness (i.e. control higher -dimension operators) just in low-energy effective field theory
- The most promising tools are shift symmetry ( $\varphi \rightarrow \varphi + c$ ) and SUSY
- Nevertheless, one needs to make assumptions about tree-level values of and **loop corrections** to operator coefficients....

$$\mathcal{L} \supset \alpha_6 \varphi^6 + \alpha_8 \varphi^8 + \dots$$

- **By contrast**, in string theory such corrections are calculable
- Furthermore, if start from string theory as *the* candidate quantum gravity theory, then for the above reasons inflation is *the* canonical way of testing it

## String theory: 'to know is to love'

- String theory solves the problems (of QFT and, in particular, of perturbative quantum gravity) in 10 dimensions:

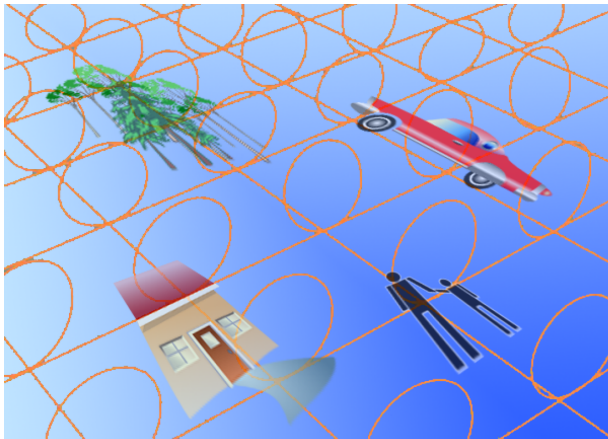


- The **divergences** at  $k \rightarrow \infty$  are now removed
- Thus, in 10 dimensions but at low energy ( $E \ll 1/l_{string}$ ), we get an (essentially) unique **10d QFT**:

$$\mathcal{L} = R[g_{\mu\nu}] + F_{\mu\nu\rho}F^{\mu\nu\rho} + H_{\mu\nu\rho}H^{\mu\nu\rho} + \dots$$

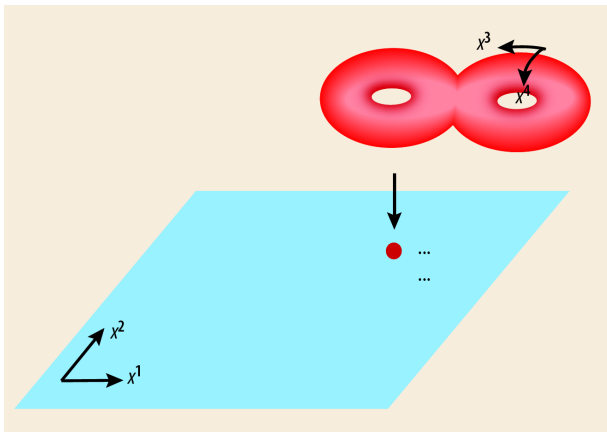
## We need to 'compactify' 6 dimensions, going from 10d to 4d

- Quite analogously, we can compactify on  $S^1$  from 3d to 2d, i.e. using  $\mathbb{R}^2 \times S^1$  as our space:



## 'Compactification' continued

- We can compactify on Riemann surfaces from 4d to 2d:



## Closer to reality:

- To go from 10d to 4d, i.e. we need 6d compact spaces
- We also want these spaces to solve Einstein's equations ( $R_{\mu\nu} = 0$ )
- Such geometries are called 'Calabi-Yau spaces' and  $\sim 10^4$  of them are known (finiteness is conjectured but not established)

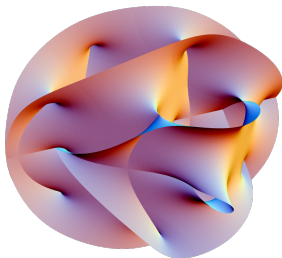
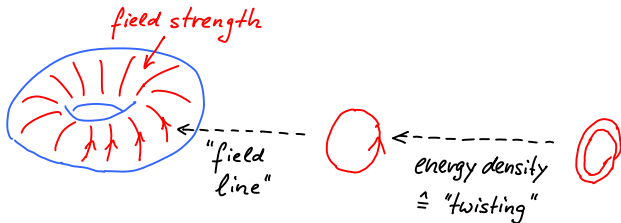


Image by J.F. Colonna

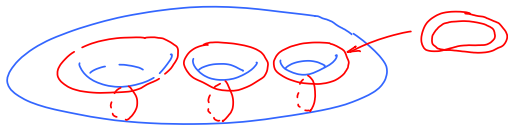
## Next crucial ingredient: Fluxes

- Fluxes are field strengths of (higher-dimensional analogues) of gauge fields, such as  $F_{\mu\nu\rho}$ ,  $H_{\mu\nu\rho}$
- They are crucial for the landscape since they stabilize the geometry and lead to  $\sim 10^{500}$  possibilities
- Simplest version of an explanation:

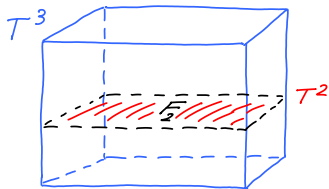


- This illustrates a flux wrapped on a 1-cycle of the torus

- Quite generally, fluxes 'live' on cycles of the compact space
- Example: several 1-cycles in 2d space



- Crucial: Higher-dimensional cycles (with fluxes) exist in higher-dimensional spaces
- Example: a 2-cycle in  $T^3$





## The string theory landscape

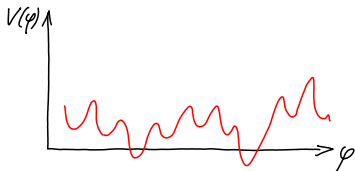
- Typical CYs have  $\mathcal{O}(300)$  3-cycles
- Each can carry some integer number of flux of  $F_{\mu\nu\rho}$ ,  $H_{\mu\nu\rho}$
- With, for example,  $N_{flux} \in \{-10, \dots, 10\}$  on gets

$$(2 \times 20)^{300} \sim 10^{500} \text{ possibilities}$$

- This is the **string theory landscape!**
- To appreciate the complexity, recall that there are only  $\sim 10^{80}$  atoms in our universe

## The string theory landscape (continued)

- Each of these geometries corresponds to a solution ('vacuum') of the same, unique fundamental theory
- Each solution has a different vacuum energy



Here  $\varphi$  corresponds to  $\{\varphi_1, \dots, \varphi_n\}$ , parametrizing the shape of the CY

Weinberg '87

Bousso/Polchinski '00

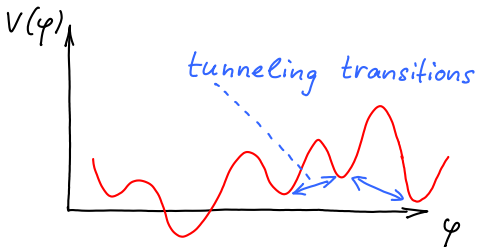
Giddings/Kachru/Polchinski '01 (GKP)

Kachru/Kalosh/Linde/Trivedi '03 (KKLT)

Denef/Douglas '04

## Populating the landscape

- Any vacuum with  $\Lambda > 0$  gives classically an eternally expanding (de Sitter) universe
- However, by a quantum fluctuation, a bubble of a different vacuum can form, which then also expands
- .... just like bubble nucleation in first order phase transitions



## Bubbles within bubbles within bubbles ....

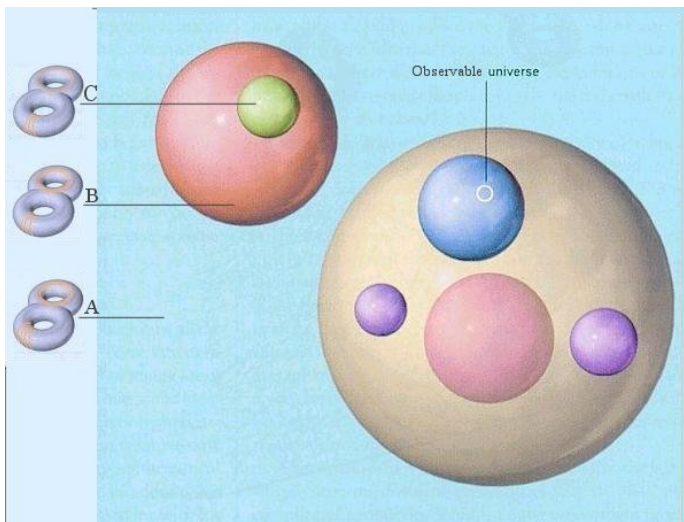
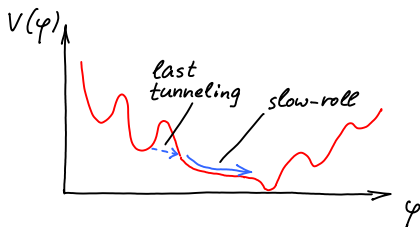


image from "universe-review.ca"

## Slow-roll inflation in the landscape

- To make our universe flat, we need a period of **slow-roll inflation** after the last tunneling event  
(...as we also argued initially purely in field theory)



- This last period of slow-roll inflation is what we observe on the CMB-sky (Cosmic Microwave Background)  
(quantum fluctuations of  $\varphi$  transform into density perturbations transform into temperature fluctuations)

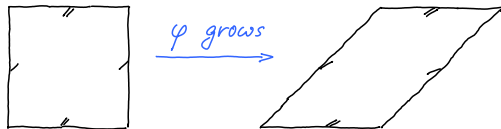
Mukhanov/Chibisov '81

## Slow-roll inflation in the string theory landscape

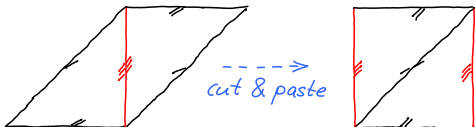
- As explained earlier, the flat piece could be short and very flat or generic, but long ( $\Delta\varphi \gg 1$ )
- Only this last option describes 'primordial gravity waves' as recently 'suggested' (???) by BICEP
- As we will now see, this feature of ' $\Delta\varphi \gg 1$ ' is extremely hard to get in string theory (chance of ruling out the landscape?)

## Why is $\Delta\varphi \gg 1$ problematic?

- The field  $\varphi$  generically corresponds to some geometric feature of the CY, e.g. the shape of a torus

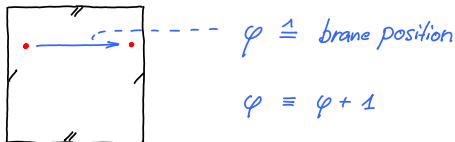


- However, after the angle of a torus has grown to  $45^\circ$ , it is secretly **the same** torus



- The problem is that this applies (more or less) to all 4d fields of a string compactification
- Another, even more obvious example arises if  $\varphi$  is a brane position. Clearly, this field is also periodic and the field space is hence limited:

Dvali/Tye '98

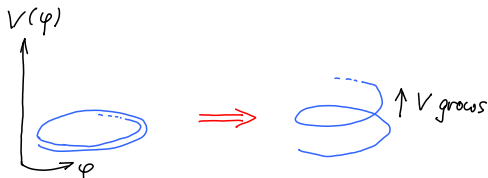


- One needs a new idea!



## Monodromy inflation

- One relatively recent such idea is to introduce a **monodromy**  
Silverstein/Westphal '08
- A **monodromy** is a change in the potential, weakly breaking the periodicity in  $\varphi$



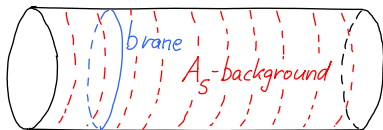
- Various concrete realizations have been discussed, especially since BICEP  
see e.g. Palti/Weigand '14  
Hassler/Lüst/Massai '14

(An alternative but related proposal is that of **'Kim/Nilles/Peloso-type models'**, not to be discussed here)

see e.g. Grimm '14

## Monodromy inflation - early models

- I will only explain a toy-model analogy to early constructions
- Let the periodic field be a Wilson line:  $\varphi = \int A_5$
- The potential is exactly flat as a result of gauge symmetry,  $A_5 \rightarrow A_5 + \partial_5 \chi$
- Flatness is broken by the presence of a brane, in the action of which  $A_5$  enters directly (rather than just  $F_{MN}$ ).



- Note: Actually, one uses not  $A_M$  but a 2-form potential  $C_{MN}$

## Monodromy inflation - early models

- One needs anti-branes, complicated non-Calabi-Yau geometries...

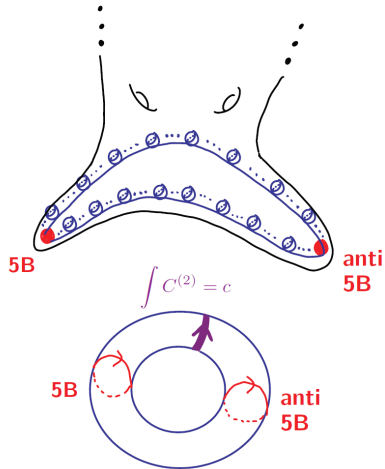
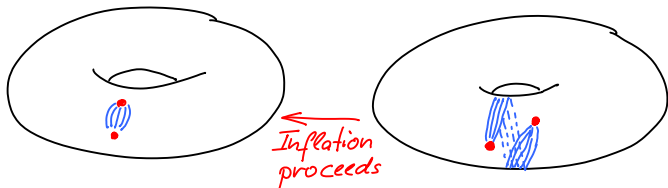


figure from McAllister/Silverstein/Westphal '08

## F-term axion inflation

- Very recently, the first suggestions have emerged how this could be realized in a quantitatively controlled way (i.e. in a 4d supergravity description, with a stabilized compact space)
  - Marchesano/Shiu/Uranga '14
  - Blumenhagen/Plauschinn '14
  - AH/Kraus/Witkowski '14
- In particular, in our suggestion inflation corresponds to **brane-motion**
- The monodromy arises from a flux sourced by the brane



## F-term axion inflation (continued)

- The strong point of these constructions is the manifest supergravity description (SUSY is broken only spontaneously, the basic geometry is still approximately Calabi-Yau, explicit calculations are feasible)
- The weak point is the required fine-tuning to make the monodromy-effect weak
- Implementing this fine tuning is subject of an ongoing debate

Blumenhagen, Herschmann, Plauschinn '14  
AH, Mangat, Rompineve, Witkowski '14

## F-term axion inflation (for the 'insiders')

- The Kahler potential is shift-symmetric (and periodic):

$$K = K(\Phi - \bar{\Phi})$$

- This situation arises e.g. in the 'large complex structure limit'
- The flux-induced superpotential breaks this symmetry (induces a monodromy):

$$W = W_0 + a\Phi$$

- The challenge is to ensure that  $a$  is sufficiently small

## Reminder of Outline

- The need for inflation / Inflation in field theory
- Why look for inflation in string theory
- The (flux-) landscape, eternal inflation and the multiverse
- Problems with large-field inflation in string theory
- Axion monodromy - early models and recent progress

## 'Conclusion'

- Inflation is developing into an interesting, quantitative playground for string theory!

Backup slides:

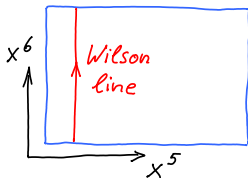


## Next-simplest version:

(For those who know about quantization of magnetic monopole charges.)

- Consider magnetic monopole in  $\mathbb{R}^3$
- For reasons of quantum mechanical consistency, the charge is quantized in units of the electron charge
- In fact, this can be seen focussing only on the field strength on an  $S^2$  surrounding this monopole
- The field strength on this  $S^2$  is 'twisted' in analogy to the Moebius strip on the previous slide
- Here, we are dealing with an  $F_{\mu\nu}$ -flux on a 2-cycle (the  $S^2$ )

## Next-simplest version, but for $S^2 \rightarrow T^2$



- With  $A_6 = \alpha x^5$  we have  $F_{56} = \alpha$
- The 'Wilson line'  $w = \int dx^6 A_6$  induces a phase  $\exp(iw)$  of the electron wave function
- In our case  $w = w(x^5)$ , which is only OK if

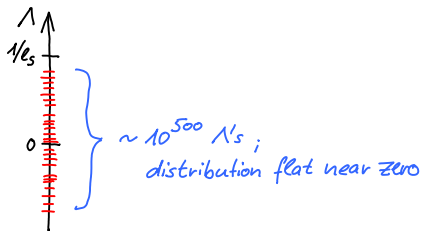
$$w(0) = w(1) + 2\pi N$$

$\Rightarrow$  Flux quantization

## The cosmological constant in the landscape

- Crucially, at least for part of the landscape, the statistical distribution of  $\Lambda = V(\varphi_{\min})$  can be calculated.

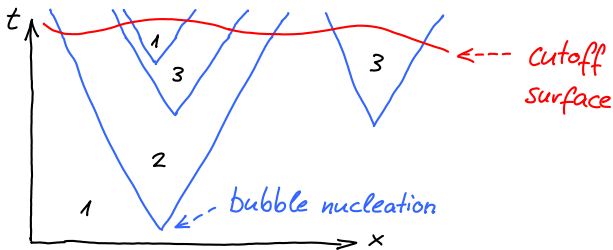
It is 'flat' in the region near  $\Lambda = 0$



- Thus, while having  $\Lambda \sim 10^{-120}$  (as is measured) is extremely unlikely, it is **known** that such vacua do exist
- One can appeal to **anthropic** arguments to explain why we find ourselves in such an 'rare' vacuum

## Bubbles within bubbles within bubbles ....

- More scientific but less pretty: A cartoon of eternal inflation in 2 dimensions



- The arbitrariness of the 'cutoff surface' is one of the faces of the measure problem – we don't know how to count and thus how to make even just statistical predictions