Quantum limited spin transport in ultracold atomic gases

Searching for the perfect SPIN fluid...

Tilman Enss (Uni Heidelberg)

Rudolf Haussmann (Uni Konstanz)
Wilhelm Zwerger (TU München)

UNIVERSITÄT
HEIDELBERG
ZUKUNFT
SEIT 1386

Technical University Darmstadt, 7 February 2013

Is an ideal fluid realized in Nature?

[Schäfer, Teaney 2009]

- flow without friction?

vanishing shear viscosity η?

some pictures are removed for copyright reasons--sorry

$$F = A\,\eta\,\frac{\partial v_x}{\partial y}$$

- kinetic theory (Boltzmann equation) for dilute gas:
  η measures momentum transport

$$\eta = \tfrac{1}{3} n\,\bar{p}\,\ell_{\rm mfp}\,, \quad \ell_{\rm mfp} = \frac{1}{n\sigma} \quad : \quad \eta \simeq \frac{\sqrt{m k_B T}}{\sigma(T)}$$

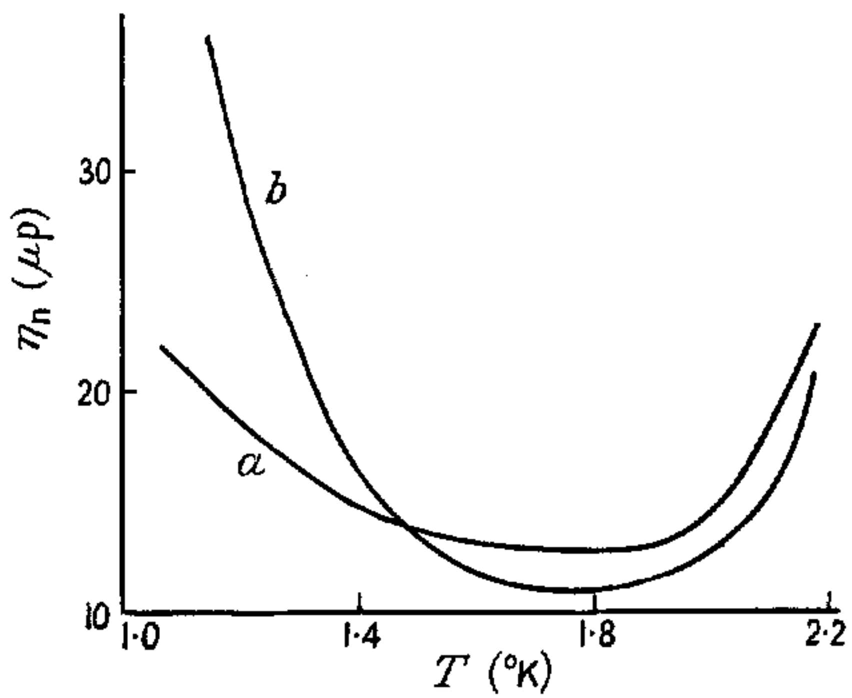
grows with T

How about a superfluid?

[Schäfer, Teaney 2009]

- superfluid helium-4: $\eta_{\mathrm{SF}} = 0$

[Heikkilä,
Hollis-Hallett 1955]



phonon contribution

$$\eta \sim T^{-5}$$

[Landau, Khalatnikov 1949]

- generically, η has minimum at strong coupling:
  universal bounds on transport coefficients?

- other sources of dissipation vanish for certain fluids
  (e.g., bulk viscosity ζ=0 in scale-invariant fluids)
  but η is always nonzero

Estimating the shear viscosity

- shear viscosity η on vastly different scales: normalize by entropy density s,

$$\frac{\eta}{s} = \# \, \frac{\hbar}{k_B}$$ (ħ indicates quantum effect)

- degenerate quantum gas: $\eta \approx \frac{1}{3} n \, p \, \ell_{\rm mfp}\,, \quad s \simeq k_B n$

  Fermi momentum $p \simeq \hbar k_F \simeq \hbar/\ell$ $\Longrightarrow$ $\frac{\eta}{s} \simeq \frac{\ell_{\rm mfp}}{\ell} \frac{\hbar}{k_B}$

  cross section limited by unitarity $\sigma \leq \frac{4\pi}{k^2} \simeq \ell^2$

  mean free path $\ell_{\rm mfp} = 1/(n\sigma) \gtrsim \ell$ (in absence of localization)

$\Longrightarrow$ $\frac{\eta}{s} \gtrsim \frac{\hbar}{k_B}$ (beyond kinetic theory: strong coupling)

Insights from string theory

- holographic duality: conformal field theory (CFT) dual to AdS_5 black hole:

shear viscosity	⟷	graviton absorption cross section (~ area of event horizon)
CFT entropy	⟷	Hawking-Bekenstein entropy (~ area of event horizon)

- specifically SU(N), $\mathcal{N} = 4$ SYM theory (no confinement, no running coupling) in strong-coupling 't Hooft limit $\lambda = g^2 N$ is dual to classical gravity:

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}$$

[Policastro, Son, Starinets 2001;
Kovtun, Son, Starinets 2005]

- conjecture of universal lower bound: **"perfect fluidity"**

Unitary Fermi gas

- two-component Fermi gas ↑, ↓ with contact interaction

$$S = \int d^d x \, d\tau \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^* \left[\partial_\tau - \frac{\nabla^2}{2m} - \mu_\sigma \right] \psi_\sigma + g \, \psi_\uparrow^* \psi_\downarrow^* \psi_\downarrow \psi_\uparrow$$

- scattering amplitude (3d)

$$f(k) = \frac{1}{-1/a - ik + r_e k^2/2}$$

- strong scattering in unitary limit

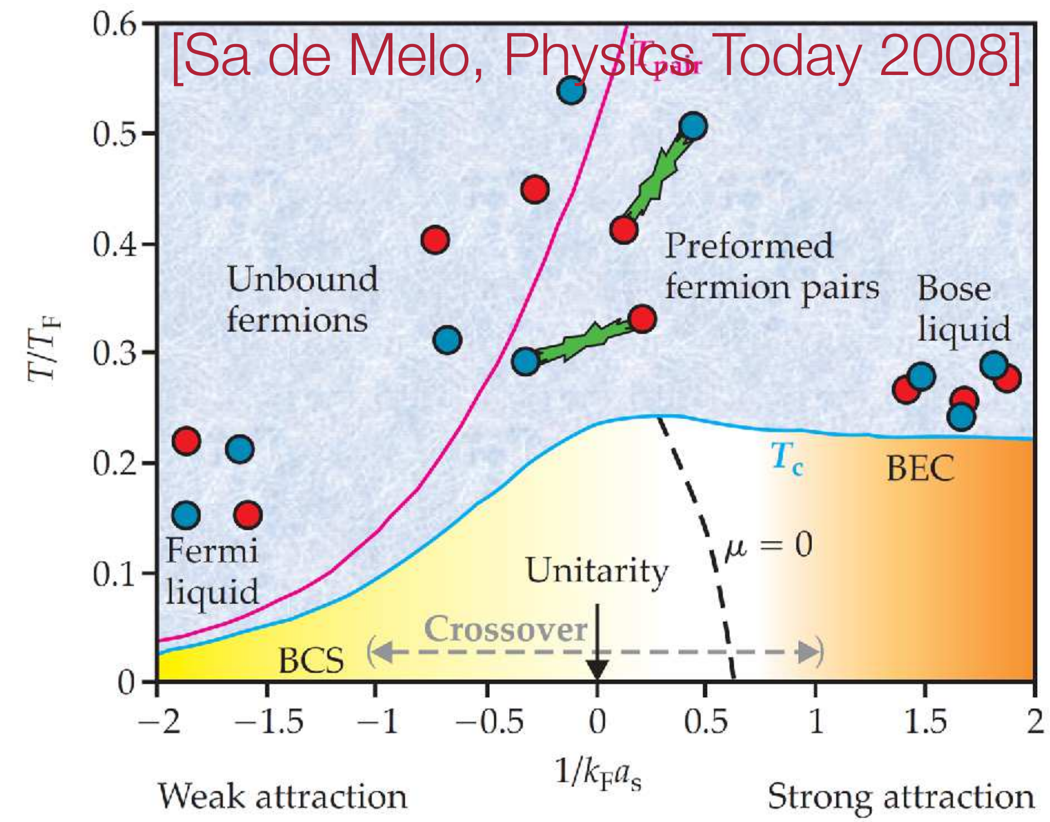
$$1/a = 0 : \quad f(k \to 0) = \frac{i}{k}$$

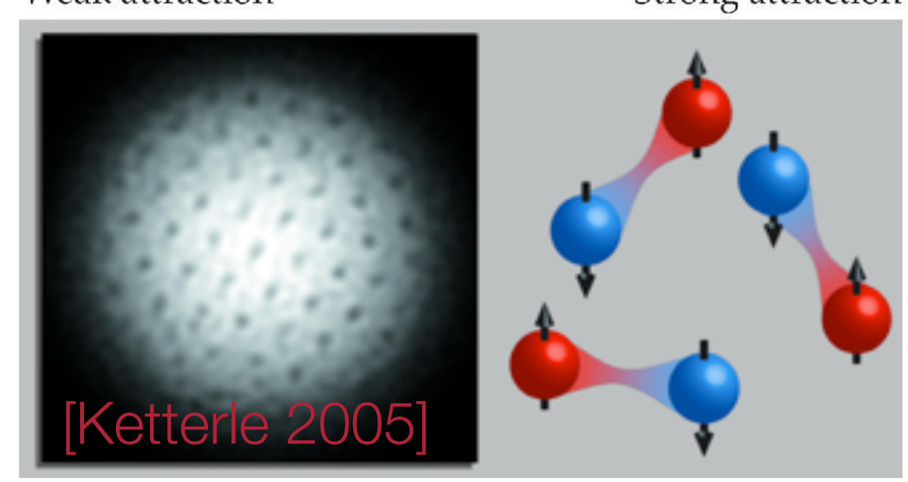
- universal for dilute system (broad resonance)

$$r_e \ll n^{-1/3}$$

- superfluid of fermion pairs below

$$T_c/T_F \approx 0.16$$ [Ku et al. Science 2012]





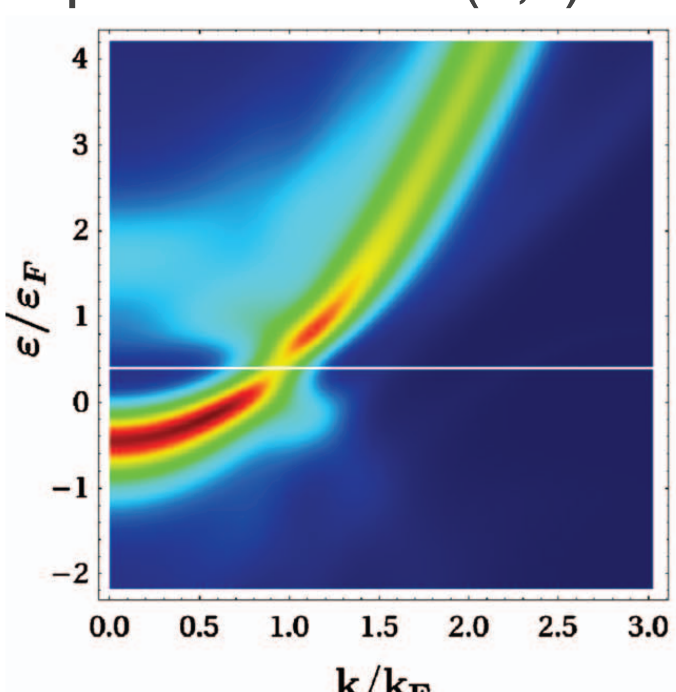
Luttinger-Ward theory

- Luttinger-Ward (2PI) computation: repeated particle-particle scattering

self-consistent T-matrix

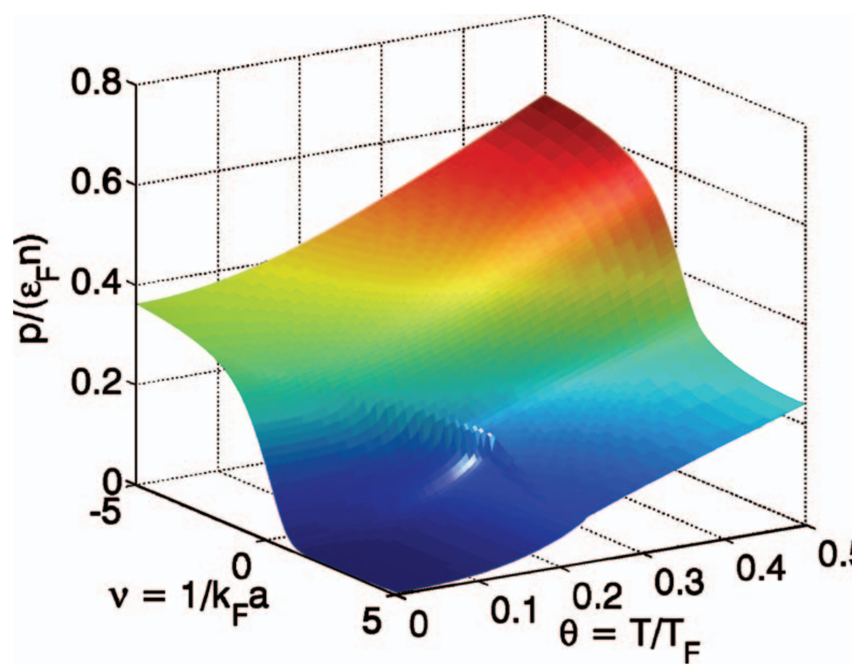
self-consistent fermion propagator
(300 momenta / 300 Matsubara frequencies)

- spectral fct. A(k,ε) at Tc



[Haussmann et al. 2009]

equation of state: pressure



[Haussmann et al. 2007]

- experiment: Tc=0.167(13), ξ=0.370(5)(8) [Ku et al. 2012, Zürn et al. 2012]
- Luttinger-Ward: Tc=0.16(1), ξ=0.36(1)

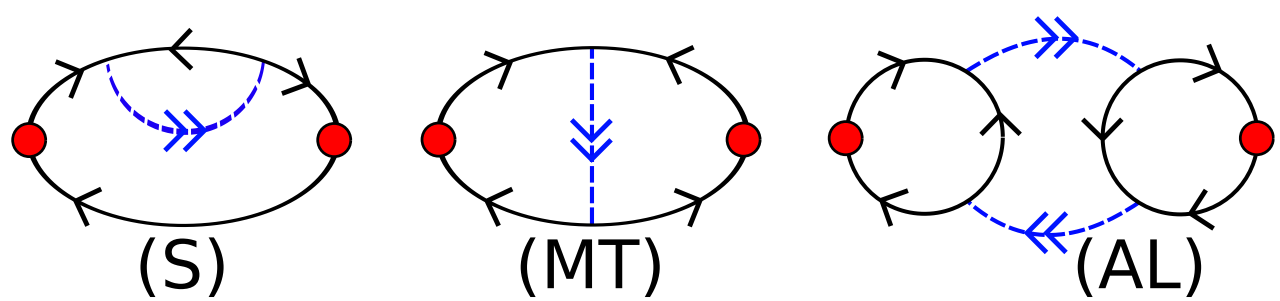
Viscosity in linear response: Kubo formula

- viscosity from stress correlations (cf. hydrodynamics):

$$\eta(\omega) = \frac{1}{\omega} \mathrm{Re} \int_0^\infty dt\, e^{i\omega t} \int d^3x \left\langle \left[\hat{\Pi}_{xy}(\boldsymbol{x}, t), \hat{\Pi}_{xy}(0,0) \right] \right\rangle$$

with stress tensor $\hat{\Pi}_{xy} = \sum_{\mathbf{p},\sigma} \frac{p_x p_y}{m} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma}$ (cf. Newton $\frac{\partial v_x}{\partial y}$)

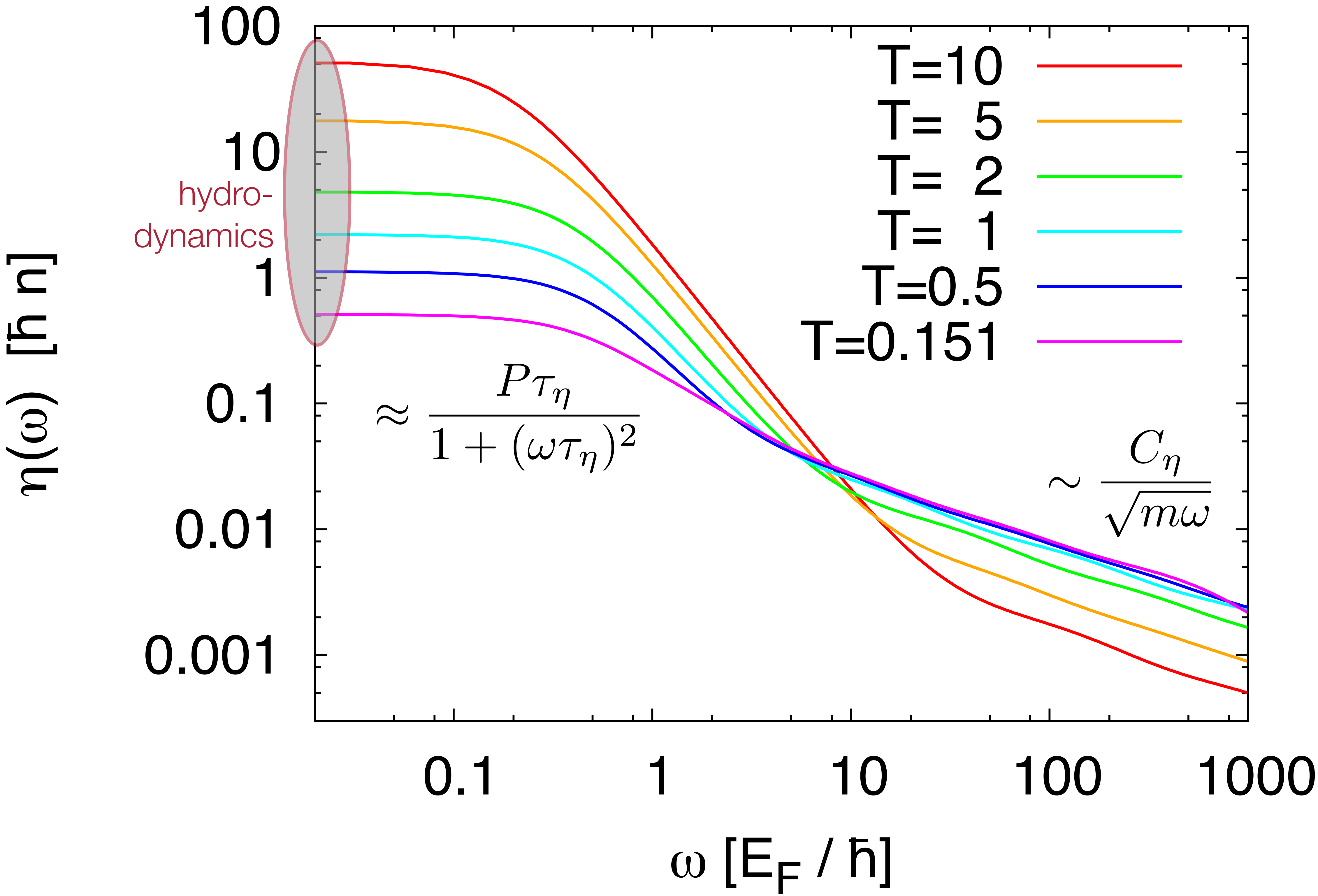
- correlation function (Kubo formula): [Enss, Haussmann, Zwerger Ann. Phys. 2011]

$\eta(\omega)$ = 

(resummed to infinite order)

- transport via fermions and bosonic molecules: very efficient description, satisfies conservation laws (exact scale invariance and Tan relations [Enss 2012])

- assumes no quasiparticles: beyond Boltzmann



Viscosity spectral function

[Enss, Haussmann, Zwerger 2011]

Contact coefficient

- generically, short-distance (UV) behavior depends on non-universal details of interaction potential

- for zero-range interaction ($r_0 \ll k_F^{-1}$) this becomes universal:
  at most two particles within distance r_0, all others far away (medium)

- two-particle density matrix for $r_0 < r \ll k_F^{-1}$:

many-body → C **few-body** → $\frac{1}{a}$

$$\int d^3\mathbf{R} \left\langle \psi_\uparrow^\dagger(\mathbf{R}+\frac{\mathbf{r}}{2})\psi_\downarrow^\dagger(\mathbf{R}-\frac{\mathbf{r}}{2})\psi_\downarrow(\mathbf{R}-\frac{\mathbf{r}}{2})\psi_\uparrow(\mathbf{R}+\frac{\mathbf{r}}{2})\right\rangle = C\left(\frac{1}{r}-\frac{1}{a}\right)^2$$

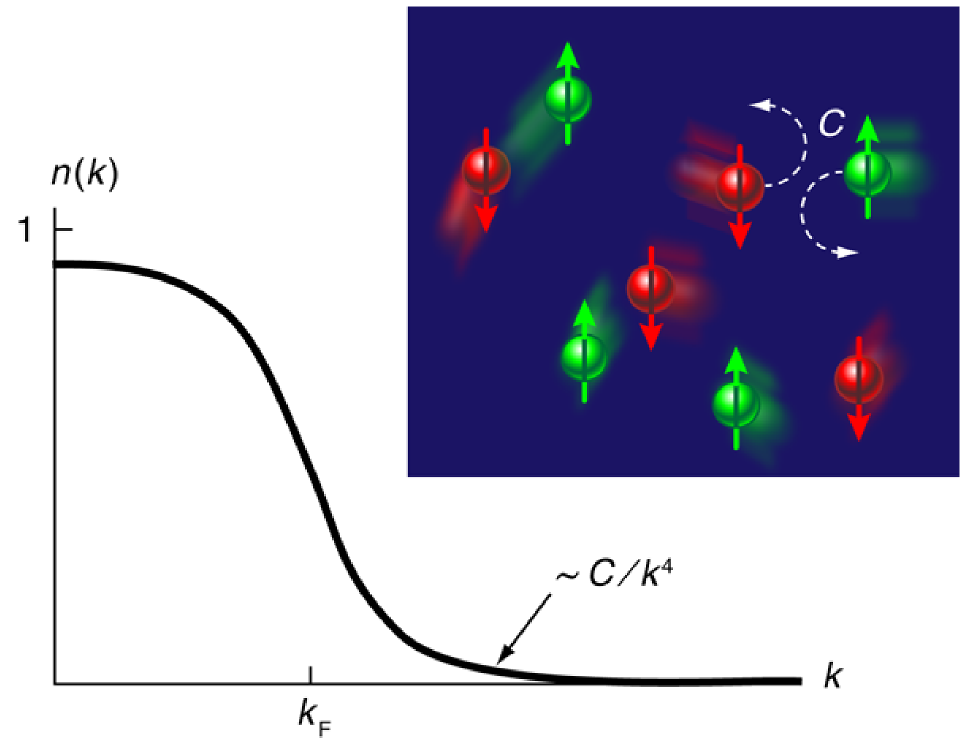
- Tan contact C: probability of finding up and down close together (property of strongly coupled medium) [Tan 2005]

Contact coefficient

- determine C:

$$\lim_{p\to\infty} n_p = \frac{C}{p^4}$$

[Stewart et al. 2010]



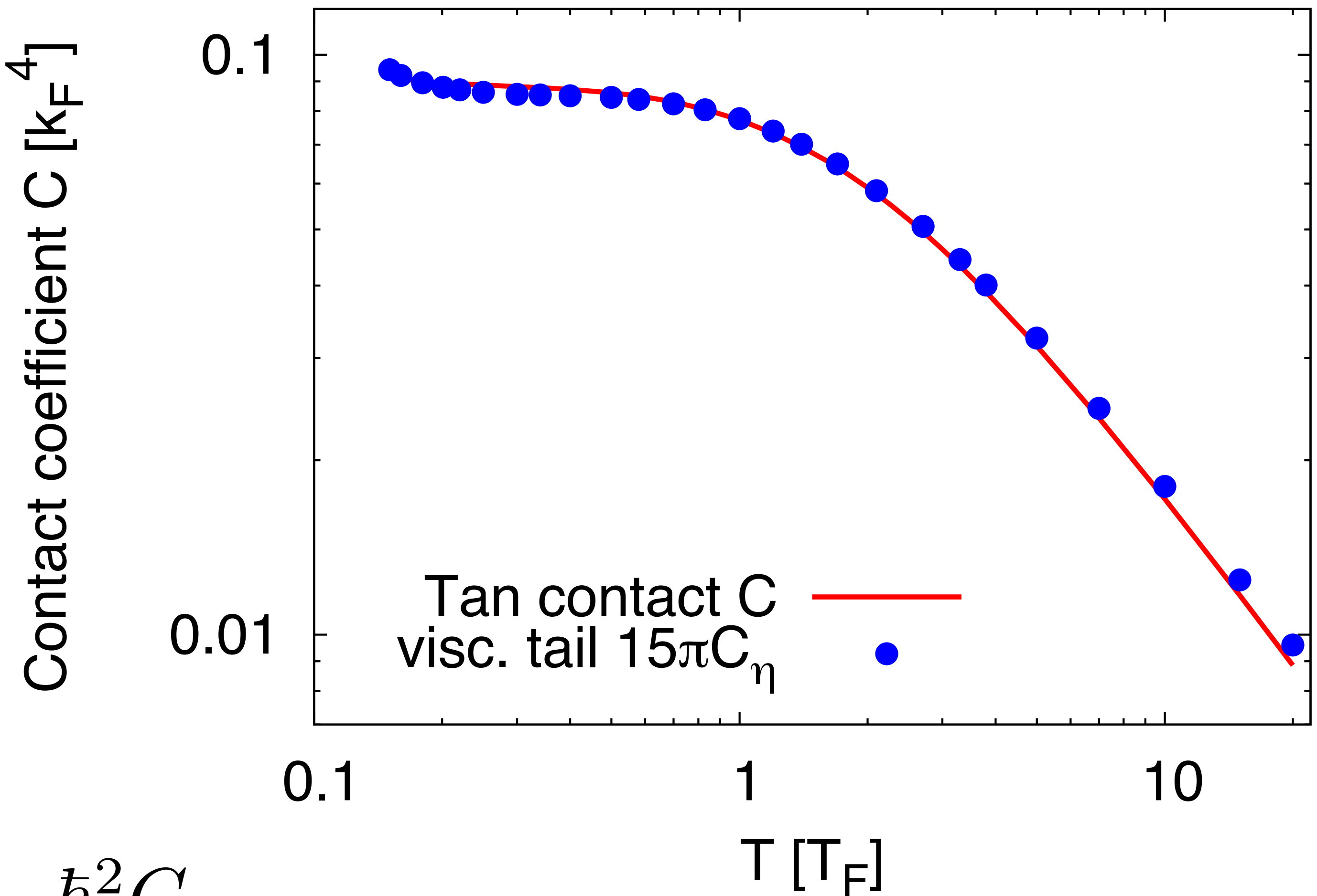
- **intuitively:** absorb external perturbation with large energy/momentum far away from coherent peak of a single particle
  ➡ need to hit 2 particles close together to give energy+momentum to both
  ➡ absorption rate ~C

- access strong coupling at arbitrary temperature via perturbation theory, predictive power (cf. Landau parameters)

Viscosity tail

- analytical high-frequency tail
  [Enss, Haussmann, Zwerger 2011]

$$\eta(\omega \to \infty) = \frac{\hbar^{3/2} C}{15\pi\sqrt{m\omega}}$$

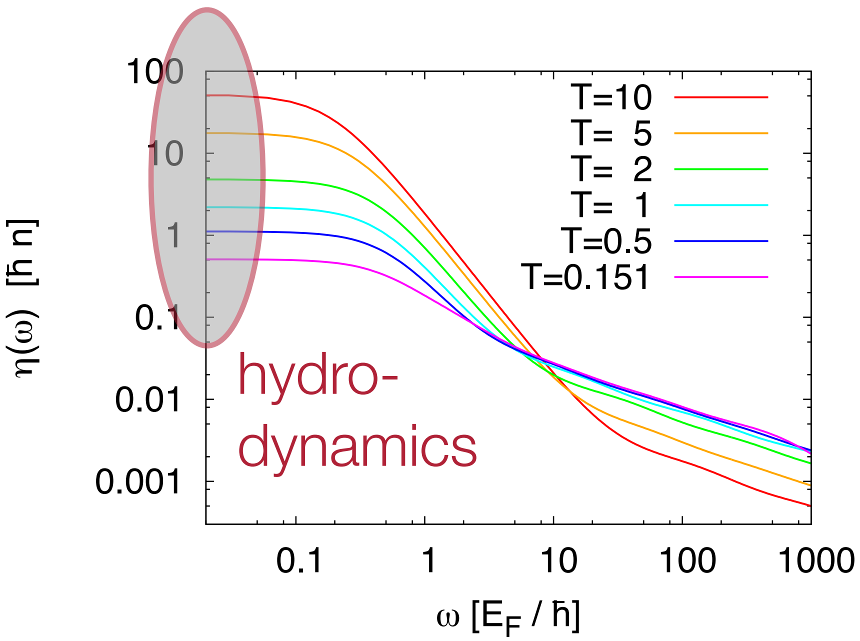


- viscosity sum rule

$$\frac{2}{\pi}\int_0^\infty d\omega\ [\eta(\omega) - \text{tail}] = P - \frac{\hbar^2 C}{4\pi m a}$$

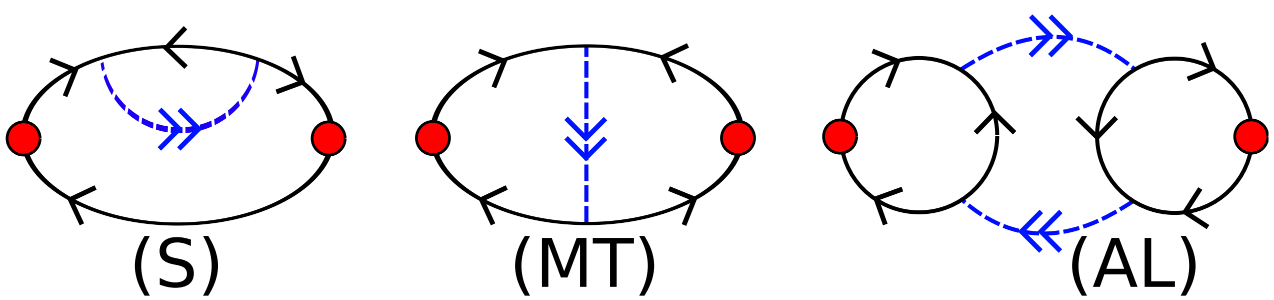
provides non-perturbative check
[Enss, Haussmann, Zwerger 2011;
cf. Taylor, Randeria 2010]

High-temperature limit



- high temperature T≫TF (virial expansion):

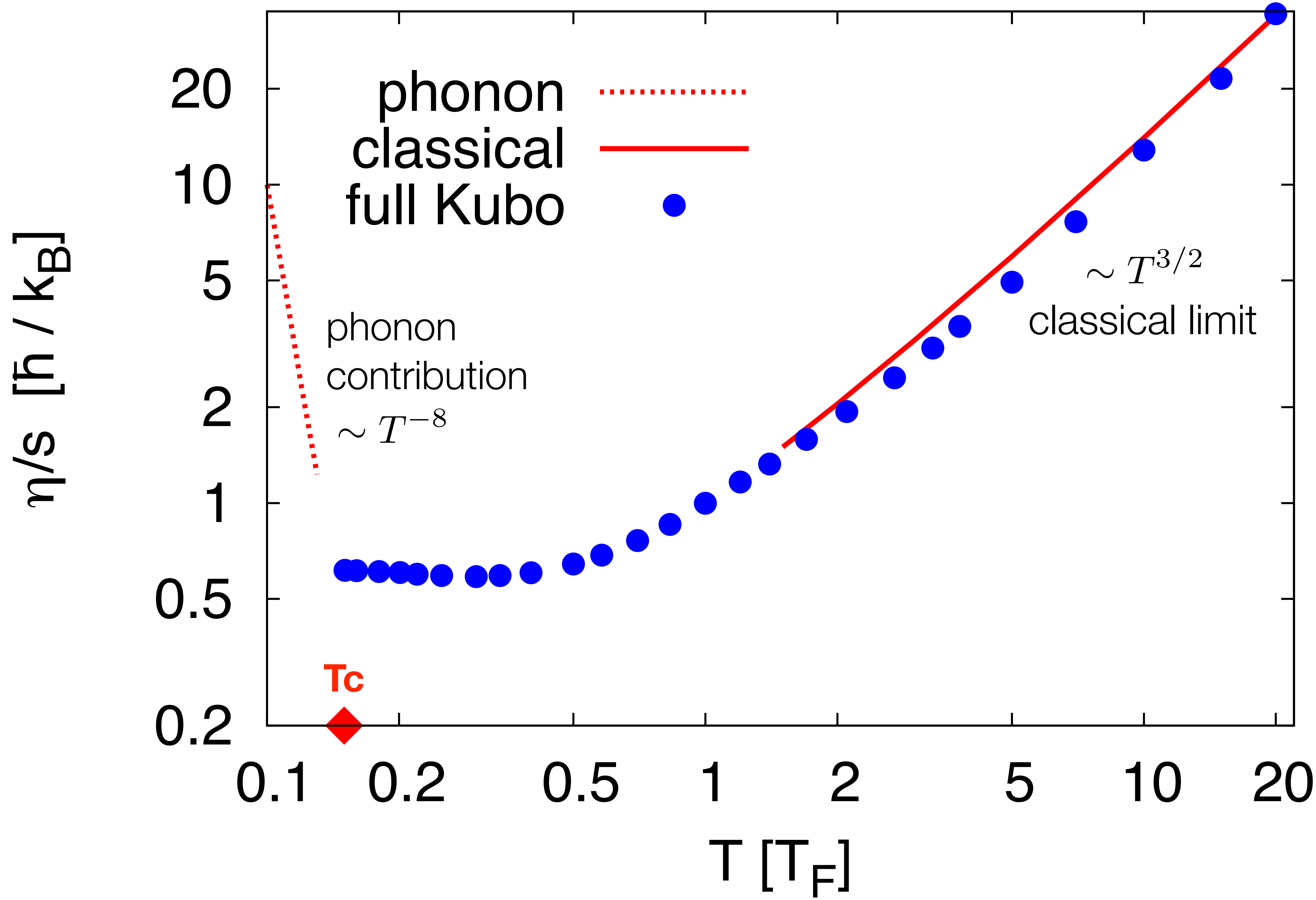
η(ω) =



$$\eta(\omega = 0) = \frac{45\pi^{3/2}}{64\sqrt{2}}\,\hbar n \left(\frac{T}{T_F}\right)^{3/2}$$

- vertex corrections crucial

- agrees **exactly** with Boltzmann result [Massignan et al. 2005]

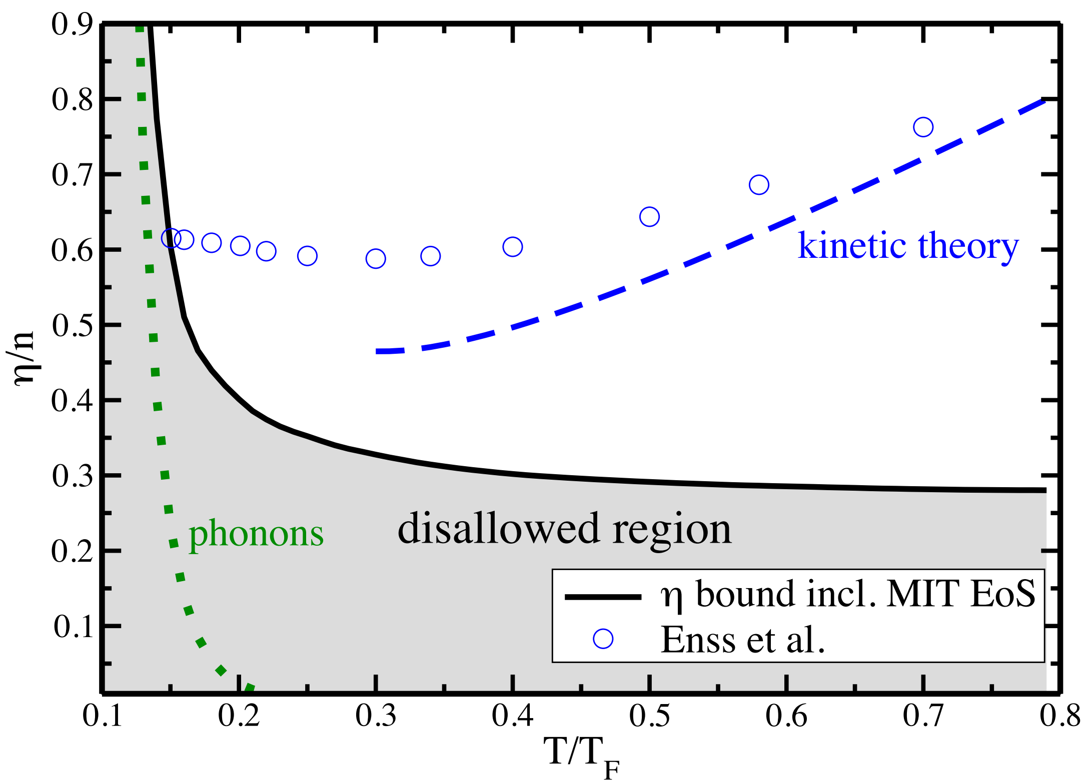


Shear viscosity/entropy of the unitary Fermi gas

[Enss, Haussmann, Zwerger 2011]

Shear viscosity bounds

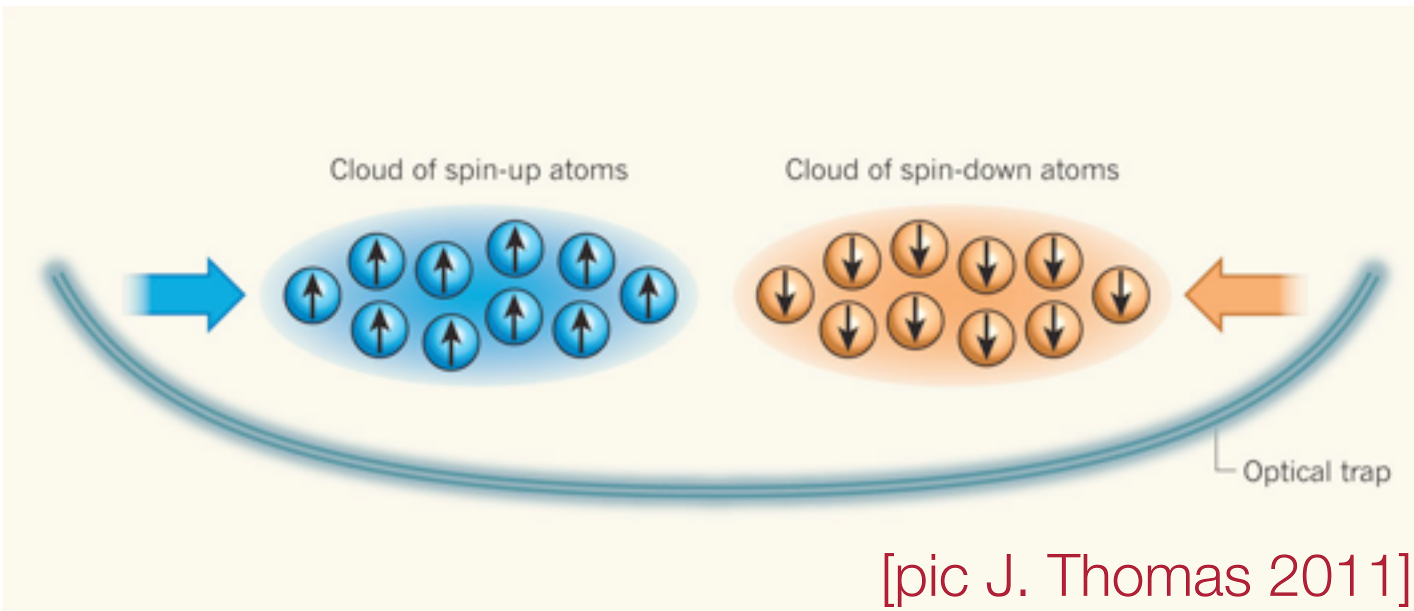
- bound from stochastic hydrodynamics: [Romatschke, Young arXiv:1209.1604]



[see also Schäfer; Bruun, Smith PRA 2007 (kin),
Enss PRA 2012 (large-N),
Wlazlowski et al. PRL 2012 (QMC),
Kryjevski arXiv:1206.0059 (ε expansion),
Schäfer, Chafin arXiv:1209.1006 (hydro)]

How about **spin** transport?

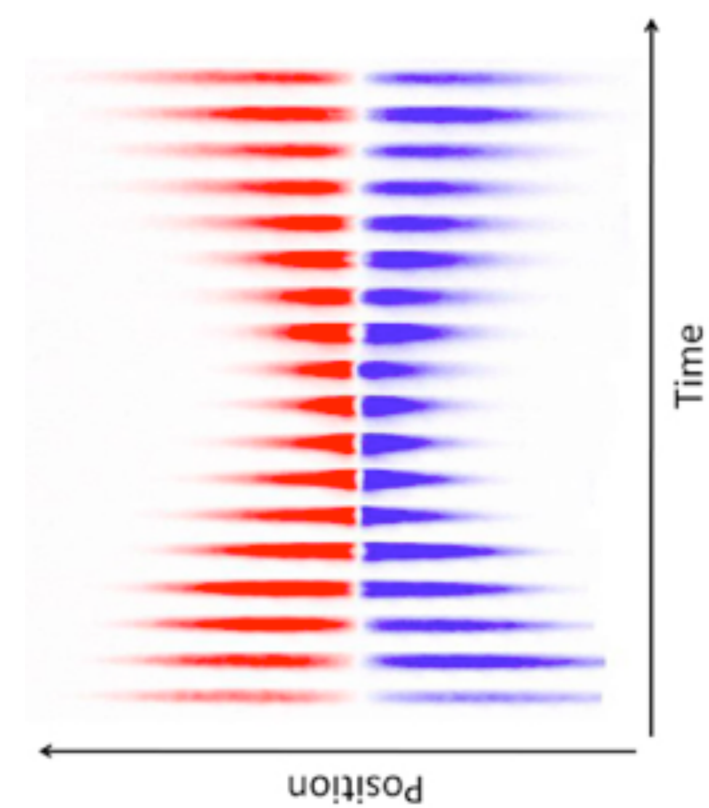
- experiment: spin-polarized clouds in harmonic trap



[pic J. Thomas 2011]

- strongly interacting gas [movie courtesy Martin Zwierlein]:

bounce!

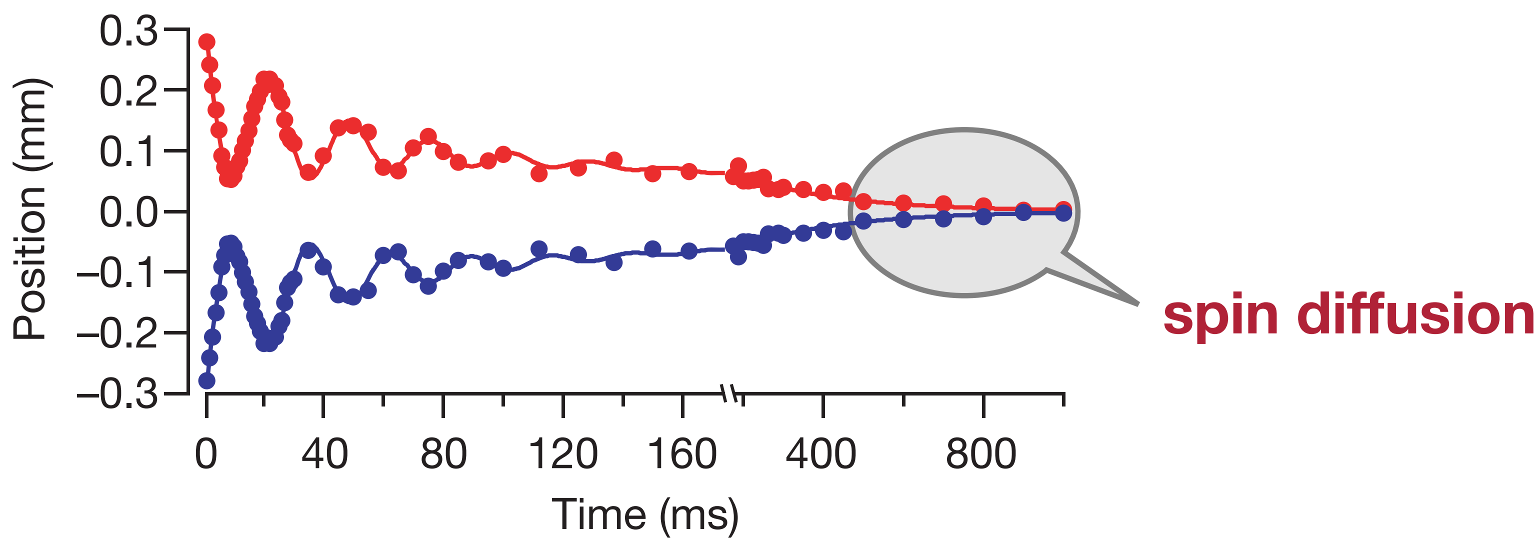


[A.T. Sommer, M.J.H. Ku, G. Roati, M.W. Zwierlein, Nature 472, 201 (2011)]

Is there a quantum bound for spin diffusion?

- scattering conserves total ⬆+⬇ momentum: mass current preserved

  but changes relative ⬆-⬇ momentum: spin current decays



- kinetic theory: diffusion coefficient $D_s \approx v\ell_{\mathrm{mfp}}$ [Sommer et al.; Bruun NJP 2011]

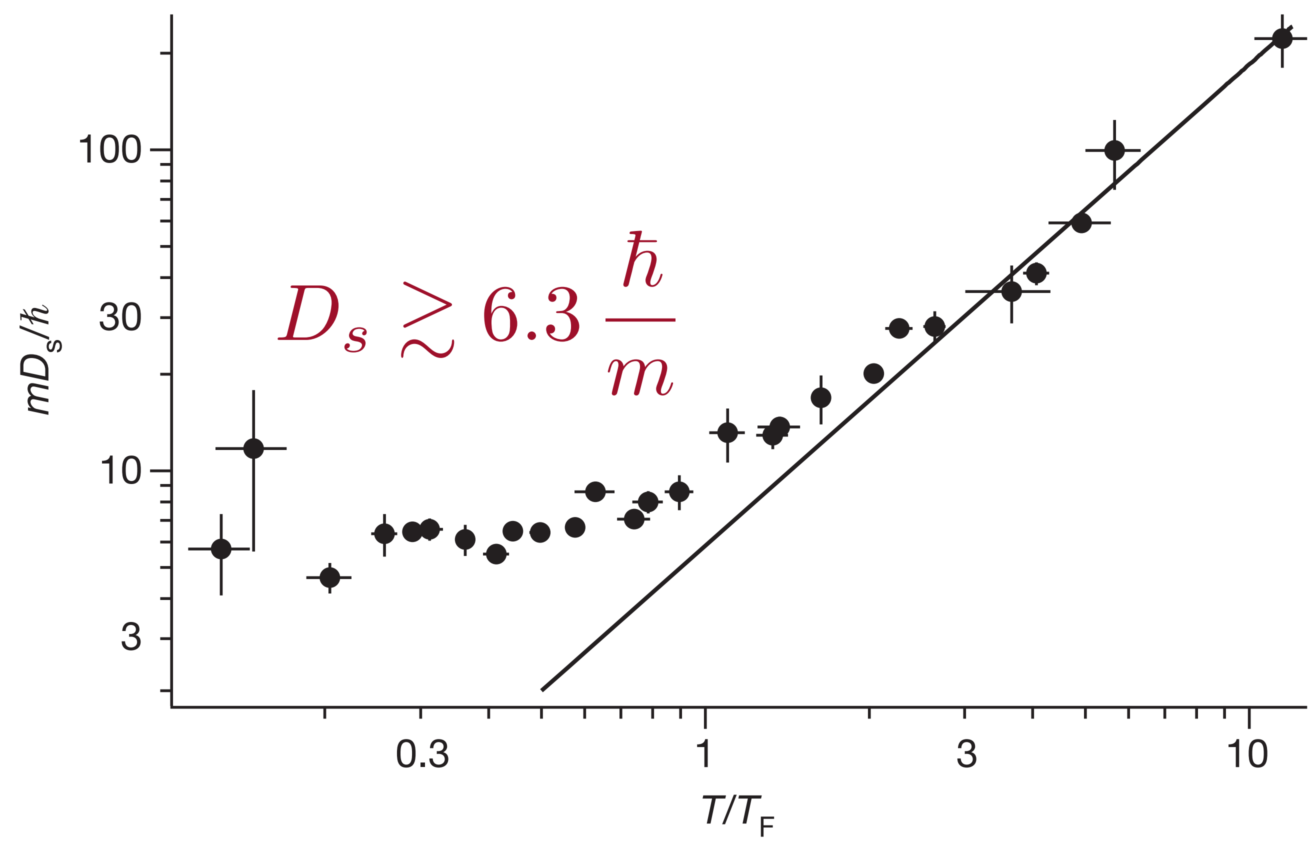
  Fermi velocity $v \simeq \dfrac{\hbar k_F}{m}$

  mean free path $\ell_{\mathrm{mfp}} = \dfrac{1}{n\sigma} \simeq \dfrac{1}{k_F}$ with cross section $\sigma \simeq \dfrac{1}{k_F^2}$ (unitarity)

$\Longrightarrow \quad D_s \simeq \dfrac{\hbar}{m}$ **quantum limit for diffusion**

Spin diffusivity

- cold atom experiment: $D_s = \frac{\text{area}}{\text{time}} \approx \frac{(100\,\mu\text{m})^2}{(1\,\text{second})} \approx \frac{\hbar}{m}$



[Sommer et al. 2011]

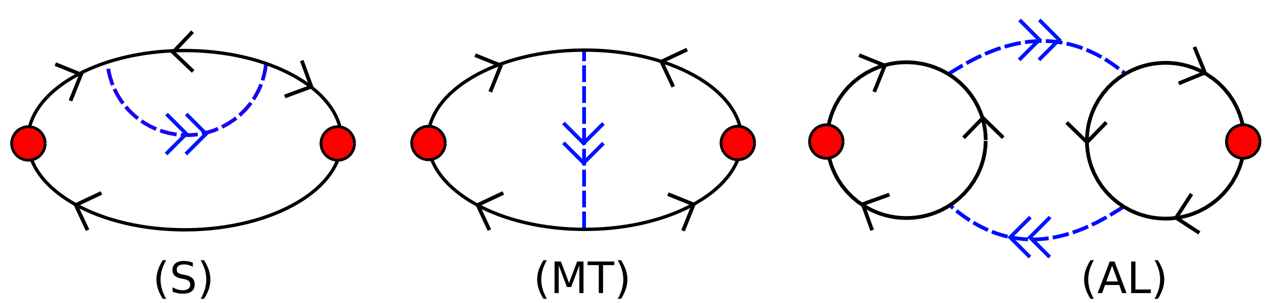
- solid state: spin Coulomb drag in GaAs quantum wells $D_s \simeq 500\,\frac{\hbar}{m}$ [Weber 2005]

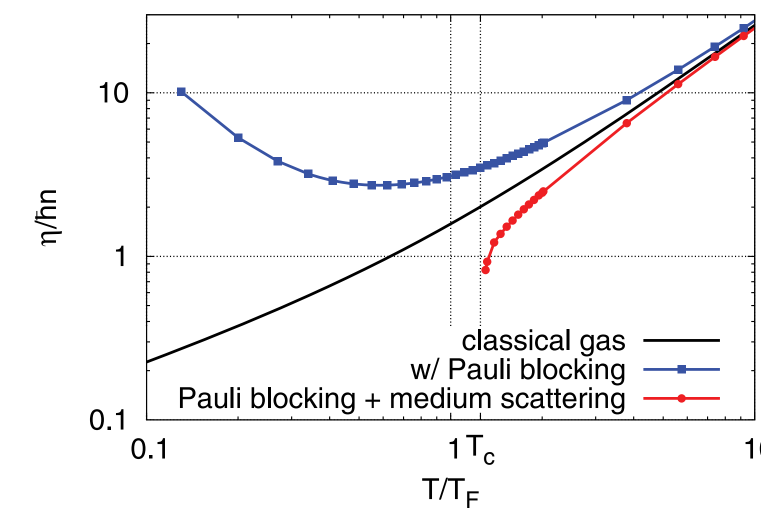
Computing the spin diffusivity

- Luttinger-Ward (2PI) theory: use Einstein relation $D_s = \dfrac{\sigma_s}{\chi_s}$

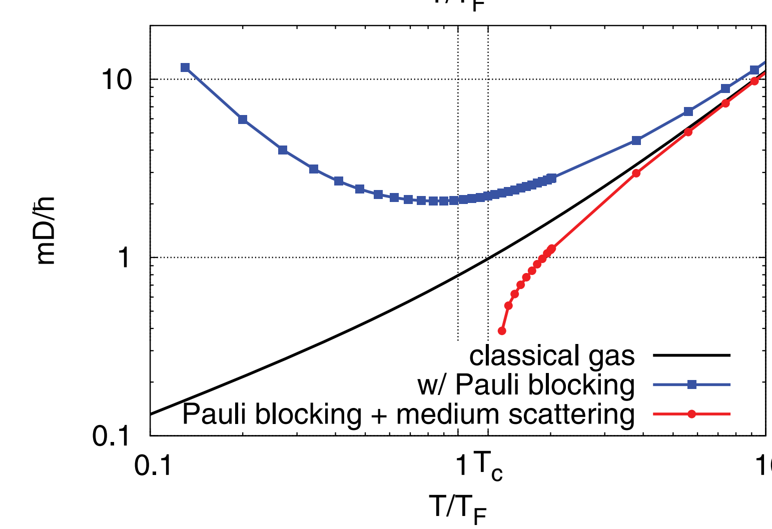
  spin conductivity $\sigma_s(\boldsymbol{q}, \omega)$ from current correlation fct. $\langle [j_\uparrow - j_\downarrow, j_\uparrow - j_\downarrow] \rangle$

- include vertex corrections to satisfy ↑, ↓ particle number conservation

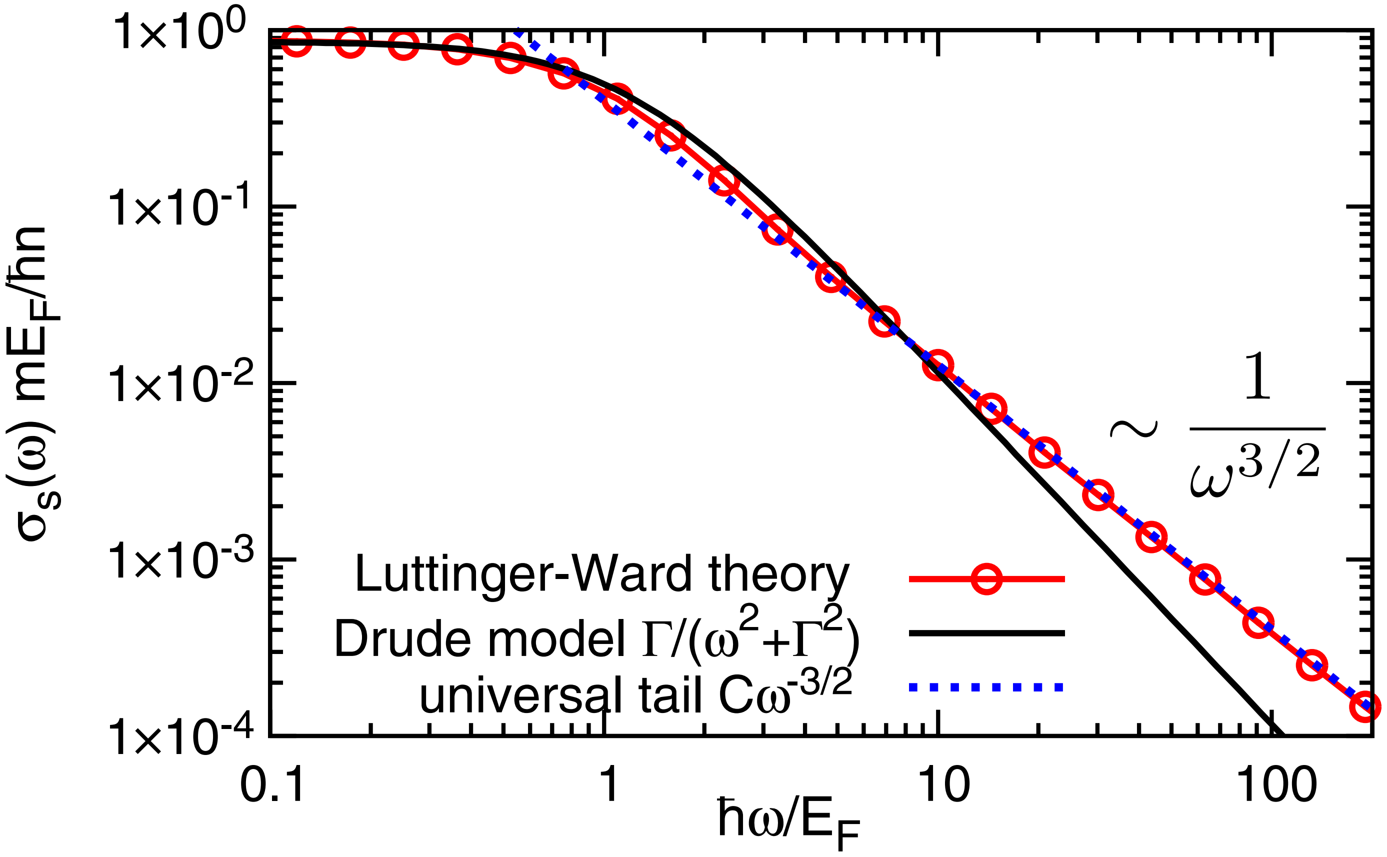




- importance of medium effects (2d):
  [Enss, Küppersbusch, Fritz PRA 2012]



Dynamical spin conductivity



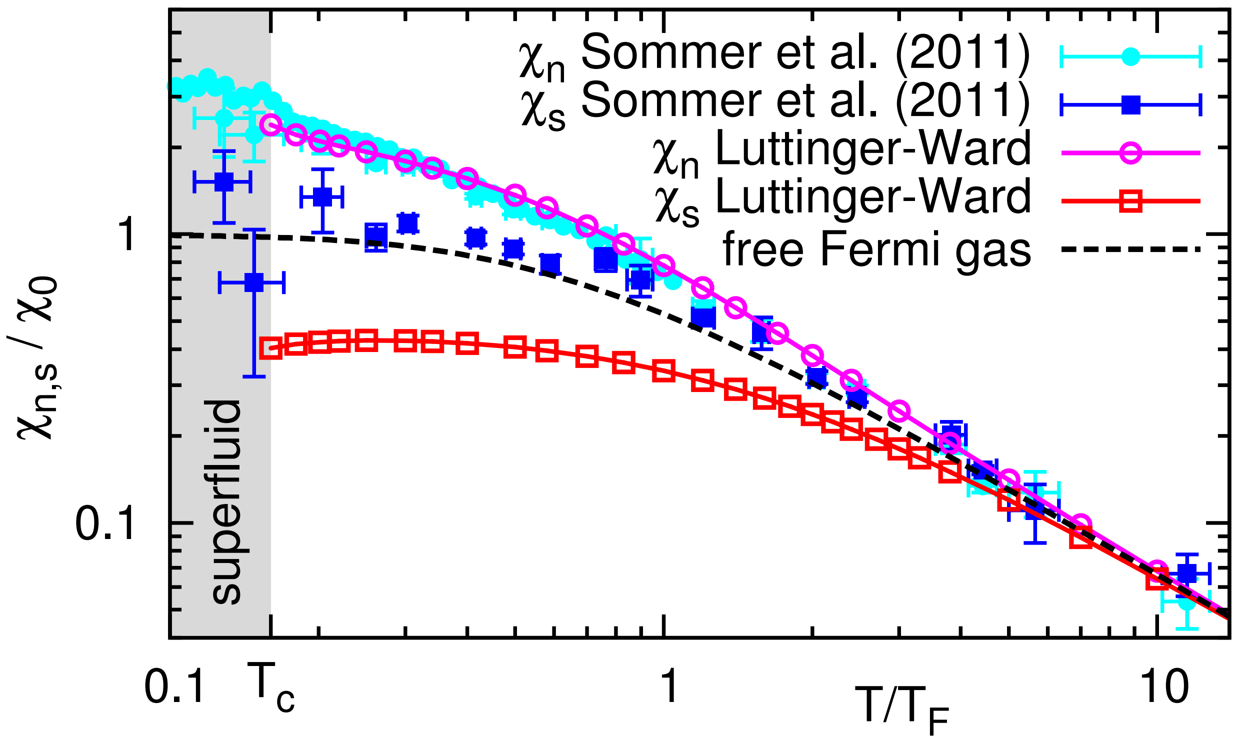
- exact high-frequency tail [Hofmann PRA 2011; Enss, Haussmann PRL 2012]

$$\sigma_s(\omega \to \infty) = \frac{C}{3\pi(m\omega)^{3/2}}$$

- satisfies spin sum rule despite tail [Enss, EPJ Spec.Topics 2013]

$$\int \frac{d\omega}{\pi}\, \sigma_s(\omega) = \frac{n}{m}$$

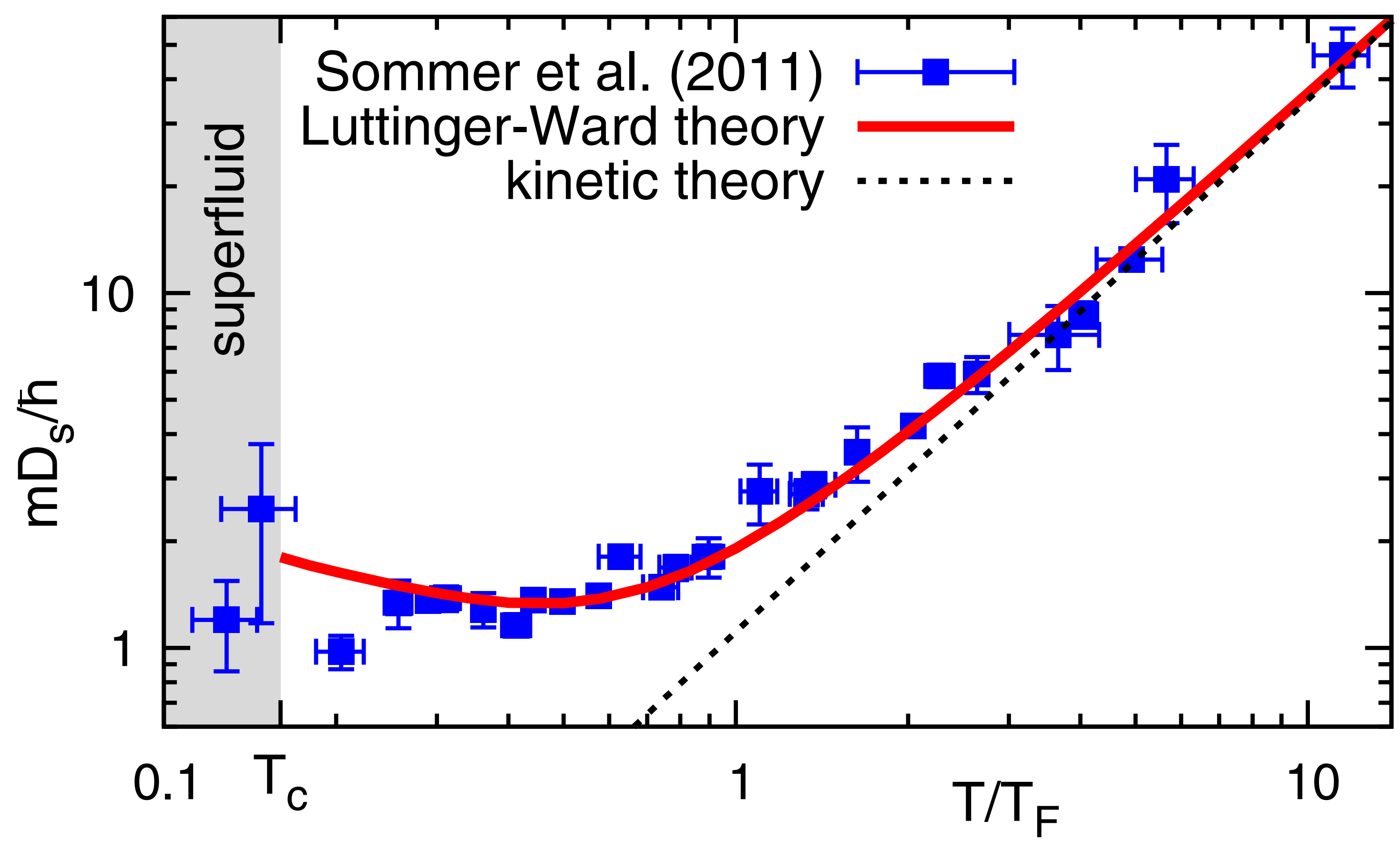
Spin conductivity and susceptibility



[Enss, Haussmann PRL 2012]

Spin diffusivity

- obtain diffusivity from conductivity, $D_s = \dfrac{\sigma_s(\omega = 0)}{\chi_s}$



(experiment rescaled from trap to infinite homogeneous box)

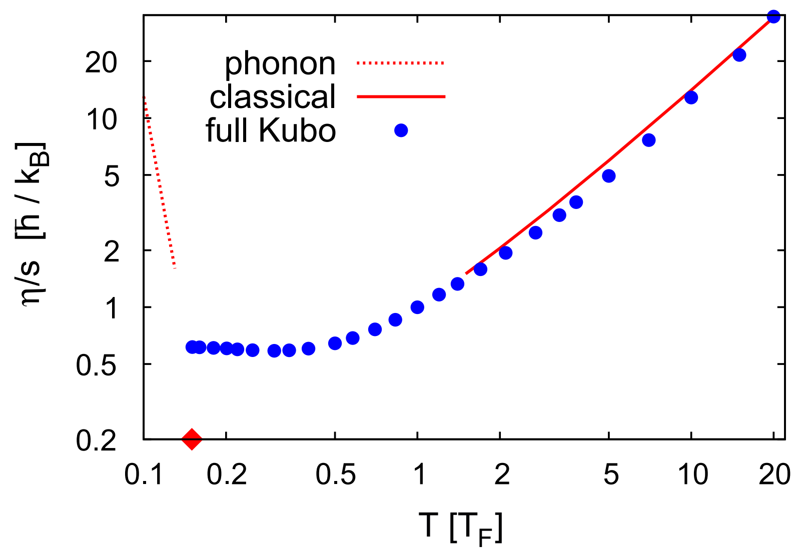
minimum $D_s \simeq 1.3 \dfrac{\hbar}{m}$

[Enss, Haussmann PRL 2012]

- recent Monte Carlo simulation for finite system: $D_s \gtrsim 0.8 \dfrac{\hbar}{m}$
  [Wlazlowski et al. arXiv:1212.1503]

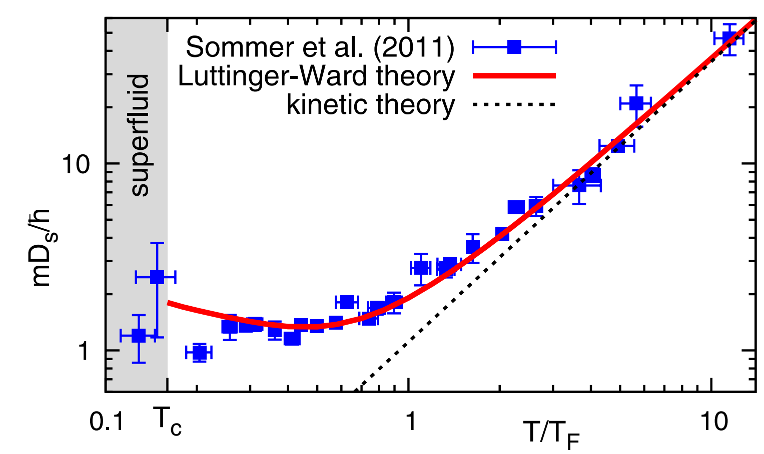
Conclusion and outlook

- universal viscosity bound:
  unitary Fermi gas most perfect non-relativistic fluid
  transport calculation beyond Boltzmann (tail, no qp)



- clouds of opposite spin bounce off each other:

- quantitative understanding of spin diffusion:
  unitary spin diffusivity $D_s \gtrsim 1.3\,\hbar/m$
  bound from holographic duality?



- challenges:
  modeling of trap, local transport measurements
  extract diffusivity from spin-resolved dynamic structure factor